Effects of Vehicles Lane-Change Manoeuvres on Traffic Breakdown and Congestion in Highways

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Abstract: *Traffic breakdown is the main cause of vehicle traffic congestion in our multi-lane roads due to highway bottlenecks such as lane-drops, on and off-ramps. In this study the three phase traffic flow theory of Kerner [1] is outlined and the nature of traffic breakdown at highway bottlenecks explained. A multi-lane macroscopic traffic flow model of Aw-Rascle type is derived from kinetic traffic flow model of Klar and Wegener [2], which expresses the lane change term explicitly. For simulation of this traffic congestion, we consider a highway with three traffic lanes that has a stationary bottleneck (on-ramp). The model equations for each lane are solved numerically using finite volume method (Godunov scheme), whereby the Euler's method was used for the source term. The results of simulation near and within the bottleneck is presented in form of graphs and space-time plots. These results indicate that vehicle lane-change manoeuvres lead to heavy traffic breakdown and congestion on the right lanes compared to the left lane adjacent to the bottleneck. This is due to the merging of vehicles from on-ramp prompting the following vehicle moving in the left lane of the highway to either slow down upon reaching the disturbance region or change to the right lanes before the vehicle reaches the merging zone.*

Keywords: Traffic breakdown, Traffic congestion, Bottlenecks, Godunov scheme, Merging zone, Lane-change manoeuvre.

INTRODUCTION

Vehicular traffic congestion is a condition on transport networks that occurs when a volume of traffic generates demand for space greater than the available road capacity, and is a major problem experienced in our roads within urban areas. It exhibits a spatiotemporal traffic pattern which is a distribution of traffic flow variables (speed and density) in space and time. One way of solving this problem is to add more lanes on the existing roadways to increase our road capacity. Sometimes this remedy is restricted by lack of space, resources, environments and bad governance. Therefore we need to have a proper understanding of empirical traffic congestion for an effective traffic management, control and organization. Traffic flow theories and models which describe in a precise mathematical way the vehicle to vehicle, vehicle and infrastructure interactions are required to explain the real cause of traffic congestion. One of the main causes of traffic congestion in our road network is traffic breakdown in an initially free flowing traffic near the bottlenecks, Kerner [1].

Traffic breakdown is described as an abrupt decrease in average vehicle speed in a free traffic flow to a lower speed in congested traffic and usually occurs at highway bottlenecks such as lane-drops, road constructions, accident area, weaving section, on and off ramps etc. This traffic breakdown is due to dynamic competition of the “speed adaptation effect” which describes a tendency of traffic towards synchronized flow and the “overacceleration effect” describing a tendency of traffic towards free flow. Traffic congestion may lead to various negative effects to motorists such as wasting time, delays in arrivals for employment and education, fuel wastage, wear and tear etcetera. However, traffic congestion has the advantage of encouraging travelers to re-time their trips early enough so that valuable road space is in full use for the most number of hours per day. Thus the need to develop macroscopic traffic flow models which describes the traffic flow dynamics by averaging vehicle density, velocity and flow rate. These macroscopic models can be used to design comfortable and safe roads. According to Kerner [1], vehicular traffic is a complex dynamic process associated with the spatiotemporal behavior of many particles systems. This is mainly due to nonlinear interactions between travel decision behavior, routing of vehicles in traffic network and traffic congestion occurrence within the road network. Normally traffic flow is considered to be either in free flow or congested state but the later exists in two different phases i.e. synchronized flow and wide moving jams. Lighthill and Whitman [2] started the macroscopic modeling of vehicular traffic by considering the equation of continuity for traffic density (ρ)and closing the equation by an equilibrium assumption on the mean velocity(u).

Later Payne [3] introduced an additional momentum equation for the mean velocity in analogy to fluid dynamics to the above mentioned model. These macroscopic models predicted that if in front of a driver traveling at a certain speed and the vehicle density is increasing but the vehicles ahead are faster, then the driver will slow down. However a common observation is that a reasonable driver will obviously accelerate when the traffic in front is moving at higher speed than he is. This inconsistency was pointed out by Daganzo [4] and was resolved by Aw and Rascle [5] who developed a new heuristic macroscopic model from kinetic equations describing the entire situation correctly. Klar and Wegener [6] derived macroscopic traffic equations from the underlying kinetic models by considering a highway with N lanes involving the vehicle interactions when changing lanes to either left or right. Ahmed [7] found that mandatory lane-change processes exhibit different behavior compared to the immediate lane-changing models of Hoogendoorn [8] who included driver behavior. That is, mandatory lane-change occurs at bottlenecks where the vehicles are forced to change to a fixed target lane and lead to traffic breakdown due to the increase of traffic demand on the road capacity. The later happens when a vehicle approaches a slower one, seeks for a sufficient gap in its target lane and change lanes immediately the gap is available. Earlier traffic flow theories and models missed the discontinuous character of probability of passing introduced in the three-phase traffic theory of Kerner [1]. Thus they could not explain the traffic breakdown at the highway bottleneck as observed in real traffic data. In this research we will use the kinetic traffic flow model of Klar and Wegener [6] which expresses the lane-change term explicitly from pure anticipation term to develop the macroscopic traffic flow model equations. According to Helbing [9], well-defined criteria for a good traffic flow model should contain only a few parameters and variables which are easy to observe, and the measured values are realistic to suit our macroscopic traffic flow model. Furthermore a good traffic model should reproduce all known features of traffic flow like localized jams and all transition states of traffic congestion and this descriptions fit our new multi-lane macroscopic traffic flow model. Thus vehicle lanechange manoeuvres can maintain free traffic flow, lead to traffic breakdown or emergence of wide moving jam near the bottlenecks and this is what our research is based on.

MATHEMATICAL ANALYSIS The Kinetic Traffic Multi-lane Flow Model

The Kinetic traffic flow model is described by use of the distribution functions of velocity of vehicles in traffic flow. We consider a Highway with N lanes numbered by□□1.....N . Let $f_{□}(□,v)$ denote a single car distribution function which describes the number of cars at x with velocity v on lane□. If $F_{□}(□,v)$denote the probability distribution in v of cars at x i.e. $f_{□}(□,v)$ □□$_{□}(x)F_{□}(x,v)$, $F_{□}{}^{□}(v^{□};h,v,x)$ denote the probability distribution in $v^{□}$ of the leading cars at distance h for cars at x with velocity v, and $Q_{□}(h;x,v)$denote the probability distribution of leading cars in h for a car at x with velocity v, then:

$f_{□}(x,v,h,v^{□})$ □ $F_{□}{}^{□}(v^{□};h,v,x)Q_{□}(h;x,v)\,f_{□}(x,v)$

Here the Kinetic equation for the distribution functions $(f_1,\cdots,f_N)$ on lanes N is obtained by finding the kinetic interaction operators, that is the Gain (G) and Loss (L) operators. Therefore,

□$t\,f$□□v□$x\,f$□□ $(GB$□ □ L□$B\,)(f$□□$1, f$□$, f$□□$1)$ □ $(GA$□ □ L□$A)\, f_{\alpha}$ $+[G_L^{+}(f$□$, f$□□$1, f$□□$2\,)$ □

(1)

L□$R\,(f$□$, f$□□$1)](1$□□□$,N\,)$ □$[GR$□$(f$□□$1, f$□$)$ □ L□$L\,(f$□□$1, f$□$, f$□□$1)](1$□□□$,1)$

Where□□$,N$ □{10 $ifif$ □□□□NN

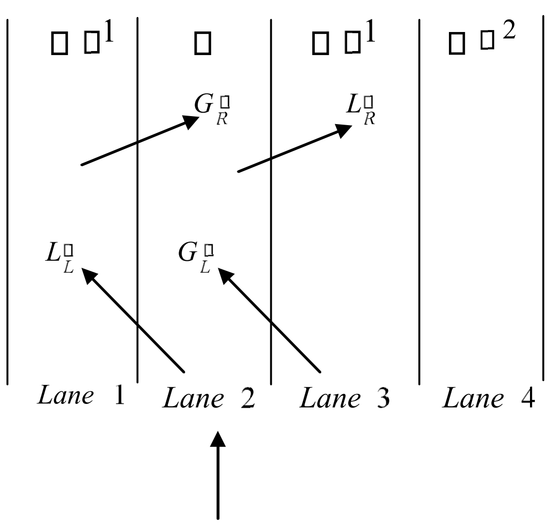
w

Taking□$_{□}$□□ $f_{□}(x,v)dv$, $f_{□}$□□$_{□}F_{□}$and $q_X(v, f_{□})$ □ $q(H_X(v),v, f_{□})$, where $H_X(v);X$ □$A,^{B}$

0

is the threshold for acceleration and braking respectively. The left hand side of the partial differential equation (1) describes the continuous dynamics of the phase-space density (PSD) due to the motion of traffic flow while the right hand side describes the discontinuous changes of this function due to lane-changing, acceleration and deceleration. Defining probability $P_Y;Y$□L,R for a lane change to either left (L) or right(R) and using the convention $P_R\,(v, f_{N□1})$ □ $P_L\,(v, f_0)$ □ 0 , then the interaction terms in equation (1) can be approximated as follows:

Remarks*: In this paper the traffic flow regulation is based on keep left lane rule for slow moving vehicles unless overtaking. The following figure 1 shows the section of the highway with the lanes under consideration during the kinetic traffic interaction operators.*



Lane under consideration and traffic direction

Fig. 1: Section of the highway showing the kinetic traffic interaction operators.

Interaction Due to Lane Changing to the Right

A vehicle will change lane to the right if the braking line is reached and a lane change is possible (probability P_R)

Gain term to the right (G_R□) is defined as:

G_R□(f□□1, f□) □ □P_R (v,□□)v□v̂□ q_B (H_B (v),□□□1)□□□1□u□□1(v)□u□□□1(v̂□)dv̂□ (2)

v□v̂□

Loss term to the right (L□$_R$) is defined as:

L□$_R$ (f□, f□□1) □ □P_R (v,□□□1)v□v̂□ q_B (H_B (v),□□)□□□u□(v)□u□□(v̂□)dv̂□

v□v̂□ (3)

Interaction Due to Lane Changing to the Left

A vehicle will change lane to the left if its follower reaches the braking line and is not able to overtake using the right lane.

Gain term to the left (G_L□) is defined as;

GL□(f□, f□□1, f□□2) □ □PL(v,□□)□1□ PR (v̂□,□□□2)□v□v̂□ qB (v̂□,□□□1)□□□1□u□□□1(v̂□)□u□□1(v)dv̂□ (4) v□v̂□

*Loss term to the left (L□$_L$) is defined a*s:

L□$_L$ (f□□1, f□, f□□1) □ □P_L (v,□□□1)□1□ P_R (v̂□,□□□1)□v □v̂□ q_B (v̂□,□□)□□□u□□(v̂□)□u□(v)dv̂□ (5)

v□v̂□

Interaction Due to AccelerationA car will accelerate if the acceleration line is reached. (a) *Gain term from acceleration (G_A□) is defined as:*

G_A□(f□) □ □□v̂□v̂□□$_A$(v,v̂□)q_A(H_A(v̂),□□)□□□u□(v̂)□u□□(v̂□)dv̂dv̂□ (6)

v̂□v̂□

(b) *Loss term from acceleration (L□$_A$) is defined as*:

L□$_A$(f□) □□ v□v̂□q_A(H_A(v),□□)□□□u□(v)□u□□(v̂□)dv̂□ (7)

v□v̂□

Interaction Due to Deceleration

A vehicle will brake if it reaches the braking line and the driver is not able to change to the right lane and if the leading vehicle cannot change to the left.

Gain term from braking interaction (G_B□) is defined as

G_B□(f□□1, f□, f□□1) □ □□ v̂□v̂□ P_B (v̂,v̂□,□□□1□□□1)□$_A$(v,v̂□)q_A(H_A(v̂),□□)□□□u□(v̂)□u□□(v̂□)dv̂dv̂□ (8)

v̂□v̂□

Loss term from braking interaction($L^{□}_{B}$) *is defined as:*

$L^{□}_{B}$ (f□□1, f□, f□□1) □|□ v□|v̂□ P_B (v̂,v̂□,□□□1,□□□1)q_B (H_B (v),□□)□□□u□(v)□u□□(v̂□)dv̂□ (9)
v□v̂□

The Macroscopic Traffic Flow Model Equations

We use the method of moments to derive macroscopic equations from the kinetic equations above. To obtain the macroscopic traffic flow equations, we multiply the inhomogeneous kinetic equation (1) by v^k, k □ 0,1 and integrating it with respect to v in the range of [0,$v_{\max}$] to get the following set of balance equations;

vmax vmax vmax

□t □vk f□dv□□x □vk□1 f□dv □ □vk{(GB□ □ L□B)(f□□1, f□, f□□1)□(GA□ □ L□A) f□□[GL□(f□, f□□1, f□□2)□
0 0 0

□ (f□, f□□1)](1□□□,N)□[GR□(f□□1, f□)□ L□L (f□□1, f□, f□□1)](1□□□,1)}dv (10) LR

Using the Gain and Loss terms interactions due to lane changing, acceleration and braking, applying the Dirac delta (δ.) function in the sense of distribution, the right hand side of equation (10) reduces to the following equations below,

Applying method of moments for $k = 0,1$:

□10(□□1,□,□□1) □{|□□□1 u|□□1 □u□□□|1 e□|□c0 1□□1□□1 □□□u□□u□□ e□□□□1c0 (1□e□□□□1c0) 1□1□□}(1□□□,1) (11)

□11(□□1,□,□□1) □{□□□|1u□□1 u|□□1 □u□□□1 e|□□□c0| 1□□1□□1 □□□u□u□□u□□ e□□□□1c0 (1□e□□□□1c0) 1□1□□}(1□□□,1) (12)

□20(□,□□1,□□2) |□{□□□1| u□□1 □u□□□1| e|□□□c0 1□□1□□1 (1□e□□□□1c0)□□□u□□u□□ e□□|□□1c0 1□1□□}(1□□□,N) (13)

□20(□,□□1,□□2) □{|□□□1u□□1 u□□1 □u□□□1 e□□□c0 1□□1□□1 (1□e□□□□1c0)□□□u□u□□u□□|e□□□□1c0 1□1□□}(1□□□,N) (14)

On the other hand the left hand side of the equation (10) simplifies to:

For $k = 0$;

□t□□□□x(□□u□) $= \varphi_1^0(\alpha - 1$,□,□□1)□□20(□,□□1,□□2) (15)

For $k = 1$;

$\square_t$(□□u□)□$\square_x$(□□u□^{2})□a(□□)□$_x$u□□□$_1{}^1$(□□1,□,□□1)□□$_2{}^1$(□,□□1,□□2) (16)

Where a(□□) is the anticipation term from drivers due to vehicles speed adaptation effect. Equation (15) and (16) are the derived macroscopic traffic flow model system of equations of Aw-Rascle type.

COMPUTATION PROCEDURE/ METHOD OF SOLUTION

We consider a highway with three lanes and an on-ramp as the bottleneck for our traffic simulation.

For a proper numerical approximation, equations (15) and (16) can be cast into:

$$\partial_t U_\alpha + \partial_x F(U_\alpha) = S(U_\alpha) \tag{17}$$

Where $U_\alpha = (\eta_\alpha, u_\alpha)^T$, $F(U_\alpha) = (\eta_\alpha u_\alpha, u_\alpha)^T$ and $S(U_\alpha)$ are vectors of conserved variables, fluxes and the source term respectively.

Given that the general initial data for the homogeneous system of equation below

$$\partial_t U_\alpha + \partial_x F(U_\alpha) = 0 \tag{18}$$

is $\tilde{U}(x,t^n)$, we can evolve the solution to a time step $t^{n+1} = t^n + \Delta t$ by use of the Godunov method (Finite volume) in the following steps:

- We assume a piecewise constant distribution of data by defining cell averages as;

$$U^n_{\alpha,i} = \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \tilde{U}_\alpha(x,t^n)\,dx$$

- We discretize the spatial domain into M cells, $C_i = [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ for $i = 1....M$ of the same size Δx. These cell averages produce the required piecewise constant distribution $U_\alpha(x,t^n) = U^n_{\alpha,i}$ for all $x \in C_i$ and $n \in \mathbb{N}$.

Considering a rectangular control volume $[x_{\alpha,i-\frac{1}{2}}, x_{\alpha,i+\frac{1}{2}}] \times (t^n, t^{n+1})$, see figure 2 below:

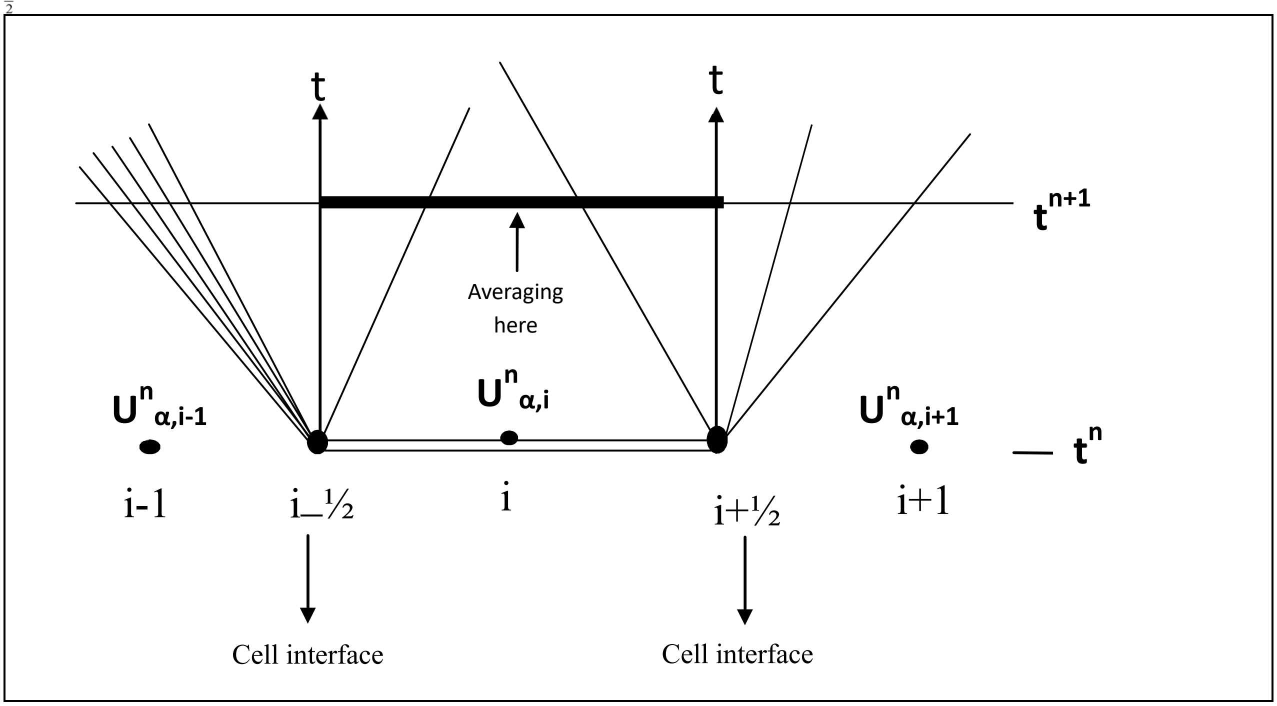


Figure 2: Typical rectangular control volume .

We obtain the required Godunov numerical scheme as:

$$U^{n+1}_{\alpha,i} = U^n_{\alpha,i} - \frac{\Delta t}{\Delta x}\left[F(U^n_{\alpha,i+\frac{1}{2}}(0)) - F(U_{\alpha,i-\frac{1}{2}}(0))\right] \tag{19}$$

Remarks: *In order to contain the interactions of the waves within the cell* C_i *during the calculations, we impose the Courant-Friedrichs-Lewy restriction (CFL condition) on time step size* $\Delta t = \frac{C_{cfl}\,\Delta x}{Max\{|u_\alpha| + \ldots,\ i = 1,2\}}$, *where* $C_{cfl} \le 1$ *is a constant called the Courant number. This is a*

condition for numerical stability where a numerical solution is unstable if the errors grow exponentially, which in turn may lead to oscillation of traffic variables with very short wavelength.
For the source term $S(U_\alpha)$, the discretized form is given by the following equations:
When $\alpha = 1$, the vehicles on lane 1 can only change to lanes 2 and 3, otherwise the other vehicles join lane 1. Thus equation (13) becomes;

□$_2^{0,(i,k)}$(1,2,3) □□2 (i,k) u2 (i,k) □u2□(i □1,k) e□□2(i,k)c0 1□□21(i,k) (1□e□□2(i,k)c0)
(20) □□11 □u1□(i □1,k)e□□2(i,k)c0 1□□11(i,k) (i,k)u (i,k)
and (14) is given by; □$_2^{1,(i,k)}$(1,2,3) □□2 (i,k)u2 (i,k)u2 (i,k) □u2□(i □1,k)e□□2(i,k)c0 1□□21(i,k) (1□e□□2(i,k)c0)
(21)
□□1(i,k)u1(i,k)u1(i,k) □u1□(i □1,k)e□□2(i,k)c0 1□□11(i,k)
When $\alpha = 2$, the vehicles on lane 2 can change lanes to right lane or left lane otherwise vehicles join lane 2. Equation (11) becomes; □$_1^{0,(i,k)}$(1,2,3) □□1(i,k)u1(i,k) □u1□(i □1,k)e□□2(i,k)c0 1□□11(i,k)

(22)
□□2 (i,k)u2 (i,k) □u2□(i □1,k)e□□1(i,k)c0 (1□e□□2(i,k)c0) 1□□21(i,k) and (12) is given by;
□$_1^{1,(i,k)}$(1,2,3) □□1(i,k)u1(i,k)u1(i,k) □u1□(i □1,k)e□□2(i,k)c0 1□□11(i,k)
(23)
□□2 (i,k)u2 (i,k)u2 (i,k) □u2□(i □1,k)e□□1(i,k)c0 (1□e□□2(i,k)c0) 1□□21(i,k)

When $\alpha = 3$, the vehicles can only change lane to the left lanes otherwise vehicles join lane 3. Therefore equation (11) simplifies to;

□1$^{0,(i,k)}$ (2,3) □□$_2$ (i,k)u_2 (i,k) □u_2□(i □1,k)$e^{□□3(i,k)c0}$ $\frac{1}{1-\rho_2(i,k)}$ (24)
And (12) reduces to;
□1$^{1,(i,k)}$ (2,3) □□$_2$(i,k)u_2 u_2(i,k) □u_2□(i □1,k)$e^{□□3(i,k)c0}$ $\frac{1}{1-\rho_2(i,k)}$ (25)

RESULTS AND DISCUSSION

In this section, we consider a highway with three lanes and an on-ramp as the bottleneck for our traffic simulations as shown in figure 3, below. Generally bottlenecks are the locations in traffic network where the road capacity is greatly reduced. At these locations, traffic demands exceed the road capacity and congestion is likely to occur, which affect the operation of the entire traffic free flow section. Thus there is a permanent speed disturbance in free flow in the vicinity of the bottlenecks where the speed is lower and the vehicle density is greater than the other part of the main road. This disturbance of free flow at the bottleneck is caused by merging of an on-ramp inflow rate (q_{on}) and a flow rate ($q_{1,in}$) on the lane adjacent to the bottleneck. This kind of traffic disturbance is referred to as deterministic disturbance and usually occurs only when the traffic flow rates are high and the average vehicle velocity is low at the on-ramp. Figure 4(a) shows the flow-density plane in lane 1, where there is a decrease in flow rate within the deterministic disturbance as the vehicle density increases at the on-ramp$(x = 0)$.

It is observed that traffic breakdown and congestion in lane 1 occurs when the vehicles from on-ramp merge with the vehicles in that lane at the bottleneck. Consequently, the aggressive drivers in lane 1 opt to change lane to the faster ones immediately they approach the traffic merging region. This is naturally a true scenario since the vehicles on lane 1 will be the ones to slow down first before the ones that are moving in the two right lanes. The flow rate in lanes 2 and 3 is sustained at the bottleneck, see figure 4 (b and c). This implies that at the bottleneck, most vehicles on the highway prefer to move in lanes 2 and 3 than in lane 1 as long as possible otherwise change lanes from left lane to the right lanes to avoid the vehicles joining the highway from on-ramp. At location $x = -10$ upstream of the bottleneck, there is a random fluctuation in flow rate with increase of traffic density as shown in figure 5 (a, b and c), that is maximum flow rate is attained at low density and vice versa. This traffic flow situation is short lived since vehicles are interacting by changing lanes from left lane 1 to the faster right lanes in the vicinity of an onramp. Therefore a transition of free flow to synchronized flow (F → S) occurs (where the flow rate is high and the average velocity is low). This (F → S) transition last for only a short period and a synchronized to free flow transition (S → F) appear. Thus, the traffic phase transition exchange is continuous at this location and complete traffic hysteresis loop, in which the upper part of the loop represents the vehicle deceleration branch in F → S transition while the lower part of the loop is the acceleration branch associated with S → F transition.

Figure 6 and 7 show the observed features of spatiotemporal congested traffic patterns that occur in the vicinity of the bottleneck. After traffic breakdown occur at the on-ramp, various patterns of synchronized flow are observed, see figure 6 and 7 (a, b and c). From these patterns, it is observed that at the bottleneck, a synchronized traffic flow in the three lanes emerges. This shows that there is a tendency towards synchronization of vehicles speeds on the highway at the bottleneck indicated by region of fluctuating low velocities, figure 7 (a). In lane 1, there is free traffic flow on the highway upstream of the on-ramp but at a distance towards the bottleneck, a moving synchronized pattern (MSP) appears. Due to the traffic freeway disturbance near the onramp by the inflow rate (q_{on}), there is an increase in vehicle density while velocity decreases in lanes 2 and 3 upstream of the bottleneck. Thus the two lanes (2 and 3) experience traffic congestion upstream of the bottleneck where the traffic queue grow at the tail while the vehicles at the head of the queue accelerate as shown in figure 6 (b and c) and 7 (b and c). However, in the three lanes downstream of the on-ramp, there is an immediate decrease in both velocity and density showing that few vehicles are able to manoeuvre out of the traffic merging region.

Table 1: Model parameters used in simulations.

c_0	0.45	$u_{□}$, *free*	□ 0.5
C_{cfl}	0.5	$u_{□}$,*synch.*	0.25
$□_{□}$, *jam*	0.9	x	x□ □ □30,10□
$□_{□}$, *free*	0.3	t	t□ □0,180□
$□_{□}$,*synch.*	0.5		

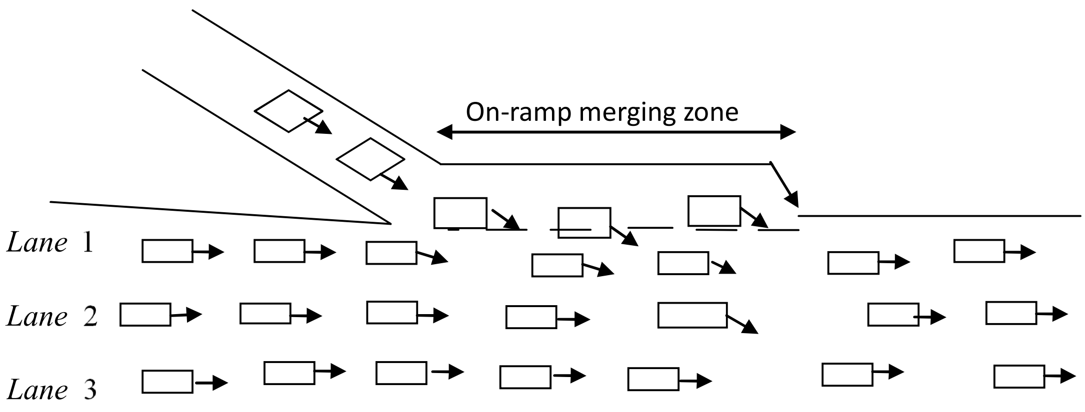
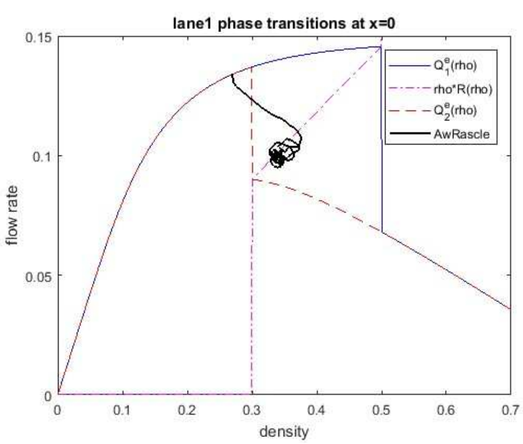
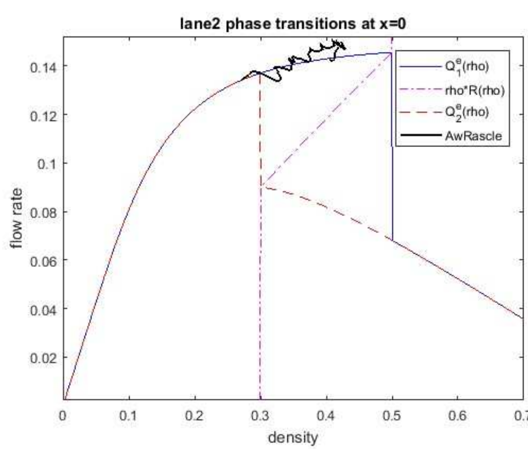


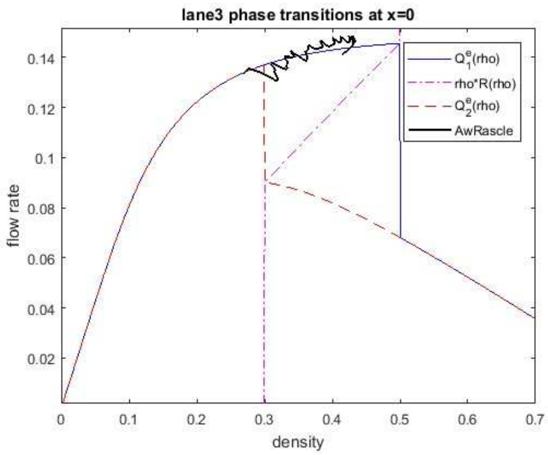
Fig. 3: Section of the highway with three lanes and an on-ramp.



(a)

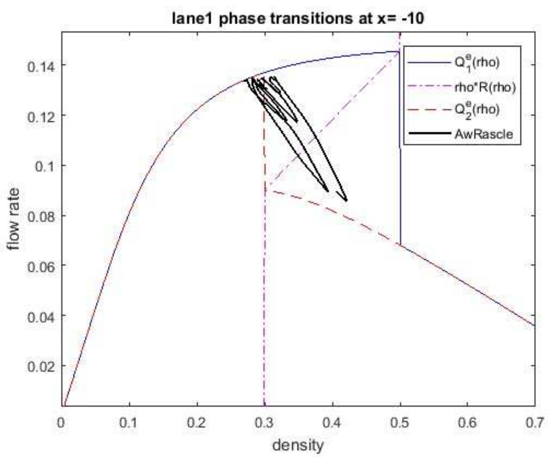


(b)

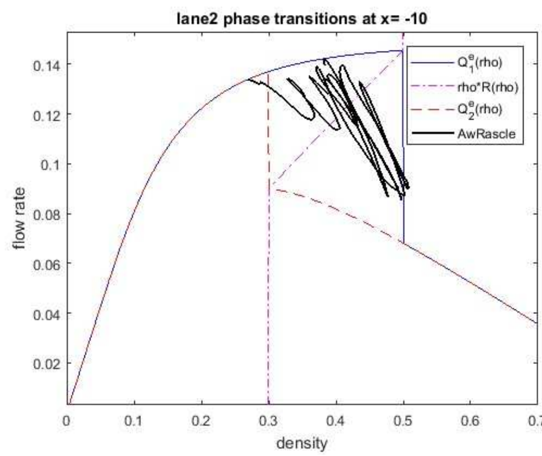


(c)

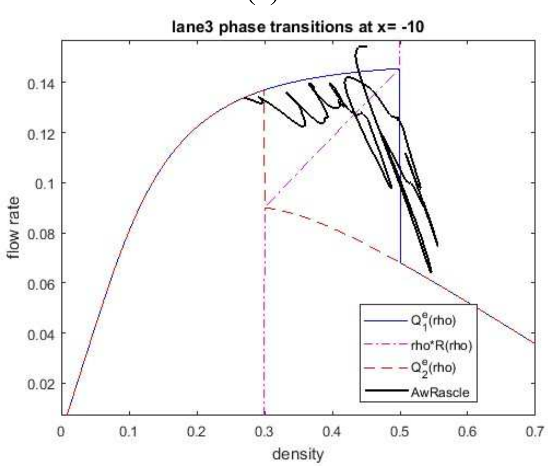
Fig. 4: Traffic flow rate-density relationship in the three lanes at location $x = 0$ of the on-ramp.



(a)

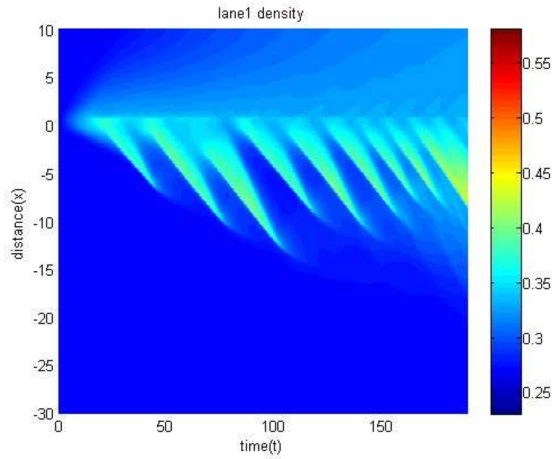


(b)

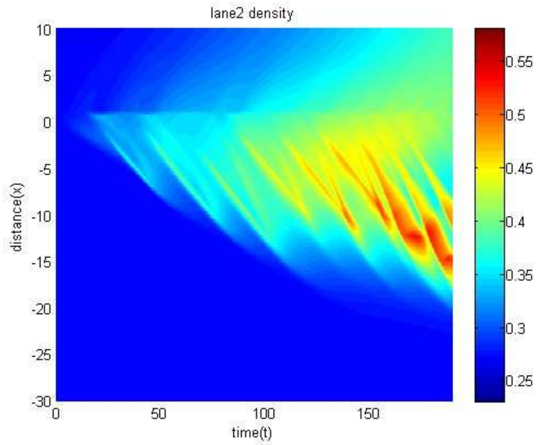


(c)

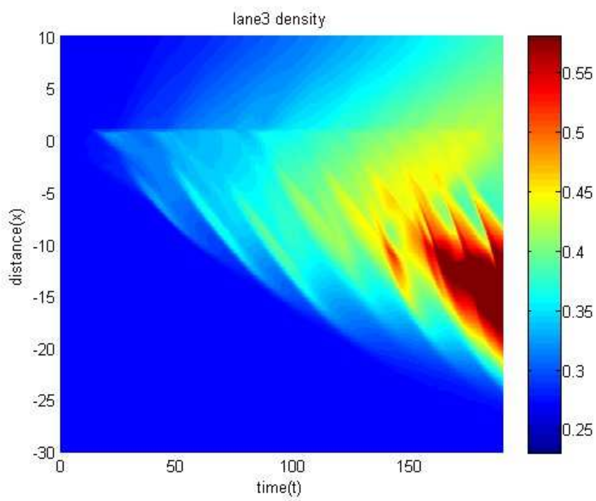
Fig. 5: Flow rate-Density relation in the three lanes at $x = -10$ upstream of the on-ramp.



(a)

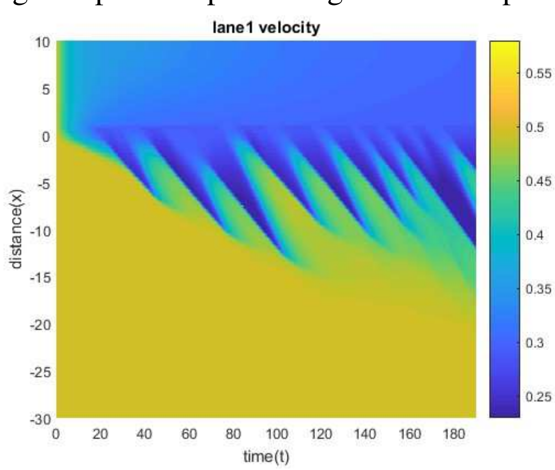


(b)

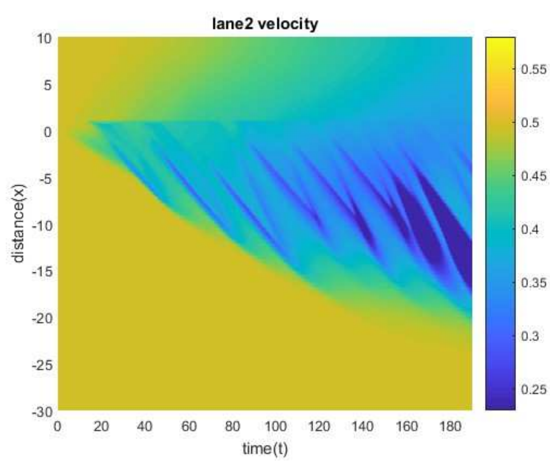


(c)

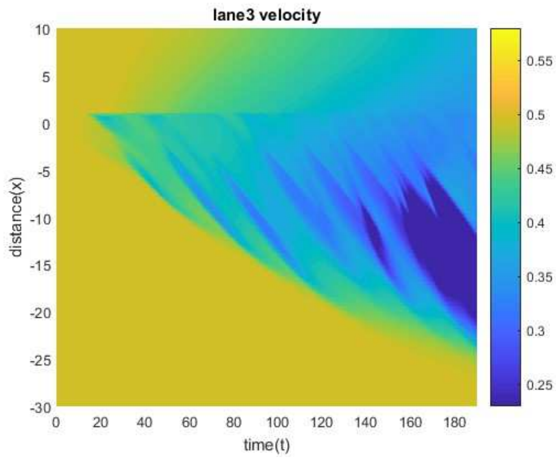
Fig. 6: Spatiotemporal congested traffic patterns of the three lanes near the on-ramp.



(a)



(b)



(c)

Fig. 7: Velocity space - time traffic patterns of the three lanes near the on-ramp.

CONCLUSION

A multi-lane macroscopic traffic flow model of Aw-Rascle type within the framework of the 3phase traffic flow theory of Kerner has been derived. This has been achieved by applying the method of moments on the kinetic traffic flow model where we obtained the kinetic interaction operators (gain and loss terms). For simulation of our traffic congestion, we have considered a highway with three lanes and an on-ramp. Finite volume method (Godunov scheme) was used to compute the numerical solutions for our traffic flow model equations. The discretized form of the source term equations are obtained for the three lanes in highway and solved using Euler's method. With these simulations near an on-ramp, the derived macroscopic traffic flow model is able to reproduce the spatiotemporal features of real traffic flow near the bottleneck. The simulations show that the initial traffic flow disturbance occurs only in the right lanes due to the merging of vehicles from on-ramp. However in contrast the disturbance can grow leading to a
transition from a free flow to synchronized one, in particular when the vehicle passing leads to the deceleration of the following vehicles in the right lanes. Therefore vehicles lane-change manoeuvre in the vicinity of an on-ramp can either lead to traffic congestion or maintenance of free traffic flow.

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