Conditional CAPM in Financial Risk Management: A Quantile Autoregression approach

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Abstract

The study aims to provide a comprehensive description of dependence pattern of a stock by studying a range of betas derived as quantiles of conditional return distribution using quantile regression based on moving window regression. We investigate predictability of various parts of the conditional return distribution in a linear, autoregressive framework. We also aim to capture a state of dependence at different quantiles of the conditional return distribution. A good (bad) state is associated with upper (lower) quantiles, thus the impact of lagged returns is different across quantiles. Our empirical findings are based on daily returns of major European stocks-sample data. Lower quantiles exhibit positive dependence with past returns while upper quantiles are marked by negative dependence. Central quantiles exhibit weak dependence. Keeping the sign of returns, we discover that positive previous day's return leads to strong positive returns with today's positive return and marked negative with today's negative return. The opposite pattern is visible for past negative returns.

Keywords and phrases: Moving window regression, CAPM, Beta, Quantile autoregression, Returns.

Introduction

- Portfolio risk assessment is an essential tool in financial risk management.

- Markowitz's mean-variance model has been critical tool in asset and fund management. It is exposed to lots of shortcoming for it relies on variance as a risk measure. Its best for elliptical and symmetric distributions.

- Most financial data display heteroscedasticity coupled with heavy tailness and skewness (stylized facts) not well captured with model based on Gaussian assumptions.

- Sharpe-Lintner-Black Capital Asset Pricing Model (CAPM) is widely used for assessing risk of cash flows from a project and determine appropriate discount rate to value projects. According to CAPM, **beta** of the cash flow w.r.t return of the market portfolio measures risk of a project.

- Quantile autoregression (QAR) allows us to explore a range of conditional quantiles, thereby exposing a variety of forms of conditional heterogeneity, and also control for unobserved individual effect. Exploring heterogeneous covariate effects within the QAR framework, offers a more flexible approach to the analysis of Stock price data than that afforded by classical Gaussian effect estimation.

- Conditional CAPM aims at testing asset pricing with time varying beta (risk premium) that can be extended to portfolio optimization techniques.

Methodology

The Econometric model

Let $\{z_t, x_t, y_t\}_{t=1}^{+\infty}$ be jointly α-mixing stationary process, where y_t is the excess return of the portfolio, and both z_t and x_t are the factors in the asset pricing model.

Define

$$y_t = \alpha(z_t) + \beta(z_t)x_t + e_t \tag{1}$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ are unknown functions of z_t. Quantile regression approach discussed by Koenker and Basset (1978) can be used to estimate $\alpha(\cdot)$ and $\beta(\cdot)$ and it is important to note that the approach is robust to heteroscedasticity, skewness and leptokurtosis.

The Unconditional CAPM model

The traditional CAMP model establishes that the expected return on any risky asset satisfies the equation

$$E(R_i) = R_f + \beta_i E(R_m - R_f) \tag{2}$$

where R_i is the return on asset i, R_f is the risk free rate, R_m is the return on the market portfolio,

and $\beta_i = cov(R_i, R_m)/var(R_m)$ is the asset's beta. We can write the equation as

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + e_i \tag{3}$$

The coefficient β_i measures the magnitude of market risk. We still can write the equation as:

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i \tag{4}$$

where ϵ_i is a random error term. The coefficient β_i measures the magnitude of market risk, and the CAPM imposes the restriction that $\alpha_i = 0$. Positive values of α_i indicate an average excess return above that predicted by the CAPM and negative values indicate an average return below that predicted by the CAPM. Analogously, the linear quantile regression model has the conditional quantile function of $(R_i - R_f)$, given $(R_m - R_f)$ as linear in covariates,

$$F^{-1}_{(R_i - R_f)|(R_m - R_f)}(\tau|(R_m - R_f)) =$$

$$\alpha(\tau) + (R_m - R_f)^\top \beta(\tau) + F^{-1}_\epsilon(\tau) \tag{5}$$

The conditional CAPM model

$$F^{-1}_{(R_{i,t}|\mathcal{F}_{\mathbf{t-1}})}(\tau|\mathcal{F}_{\mathbf{t-1}}) = \gamma_0(\tau) + \beta^\top_{i,t-1}\gamma_1(\tau) + F^{-1}_{\epsilon_i}(\tau)$$

(6)

where $\beta_{i,t-1}$ denotes the ith unconditional beta. The coefficient $\gamma_{1,t-1}(\tau)$ may be interpreted as quantile specific autoregressive coefficient representing the conditional market risk premium at some given $\tau \in \Gamma$, and is the focus of this study. We assume the model eq (6) has zero quantile and unit scale hence we represent the DGP as

$$F^{-1}_{(R_{i,t}|\mathcal{F}_{\mathbf{t-1}})}(\tau|\mathcal{F}_{\mathbf{t-1}}) = \gamma_0(\tau) + \beta^{\top}_{i,t-1}\gamma_1(\tau) \quad (7)$$

which can be solved by applying QR as in Koenker and Basset (1978) where

$$\gamma(\tau) = arg \min_{\gamma_j \in \mathbb{R}^{\mathbf{2}}} \sum_{t=1}^{T} \rho_\tau \left(R_{i,t} - \gamma(\tau) x^{\top}_{i,t-1} \right) \quad (8)$$

$$for\ j = 0, 1$$

We can extend the model to account for the size of a lagged return as:

$$F^{-1}_{(R_{i,t}|\mathcal{F}_{\mathbf{t-1}})}(\tau|\mathcal{F}_{\mathbf{t-1}})$$

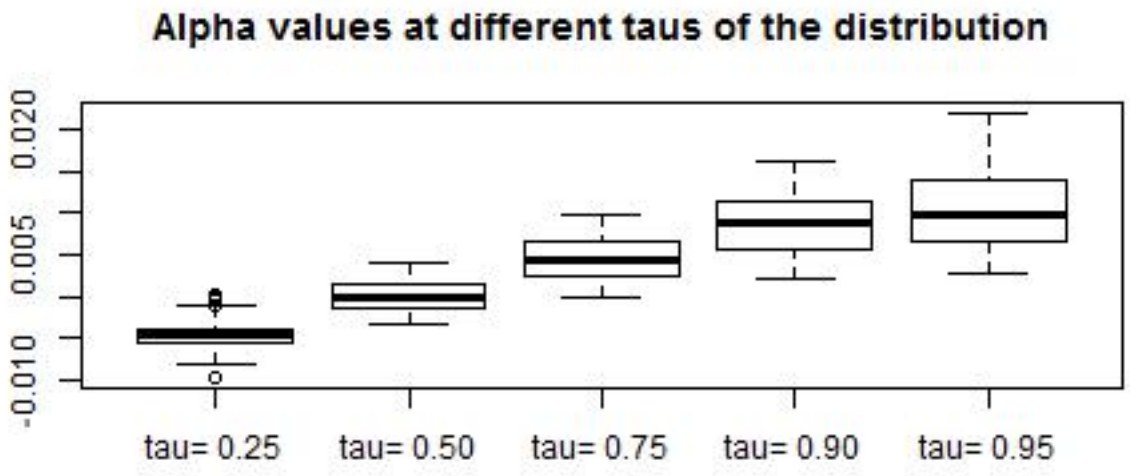
$$= \gamma_0(\tau) + \beta^{\top}_{i,t-1}\gamma_1(\tau) + \alpha_i(\tau)\beta^{\top}_{i,t-1} I\left(\left| \beta^{\top}_{i,t-1} \right| > r^q \right) \quad (9)$$

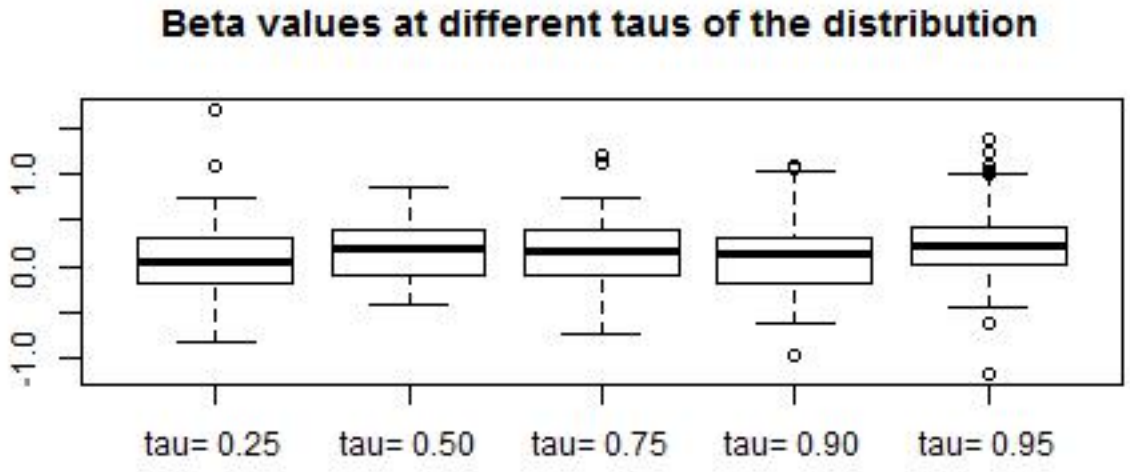
where the indicator variable $I\left(\left|\beta_{i,t-1}^{\top}\right| > r^{q}\right)$ is equal to one if the β of stock at i lag in period t-1 exceeds a certain threshold r^{q} and zero otherwise. We can choose the value of r^{q} to be the 95% quantile to assess the influence of both extreme positive as well as extreme negative of a previous day's return. We can also extend (7) to capture the role of the sign of previous period's return given by:

$$F^{-1}_{(R_{i,t}|\mathcal{F}_{\mathbf{t-1}})}\left(\tau|\mathcal{F}_{\mathbf{t-1}}\right)$$

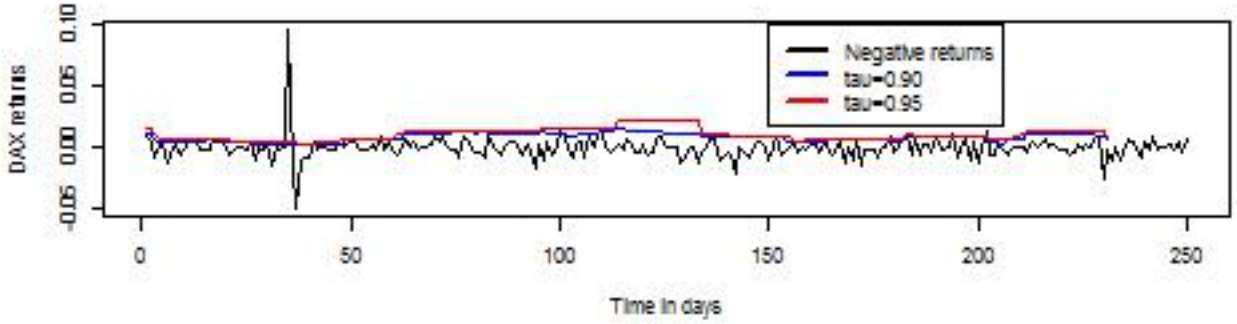
$$= \gamma_0(\tau)+\beta_{i,t-1}^{\top}\gamma_1(\tau)+\alpha_i(\tau)\beta_{i,t-1}^{\top} I\left(\left|\beta_{i,t-1}^{\top}\right| < 0\right) \tag{10}$$

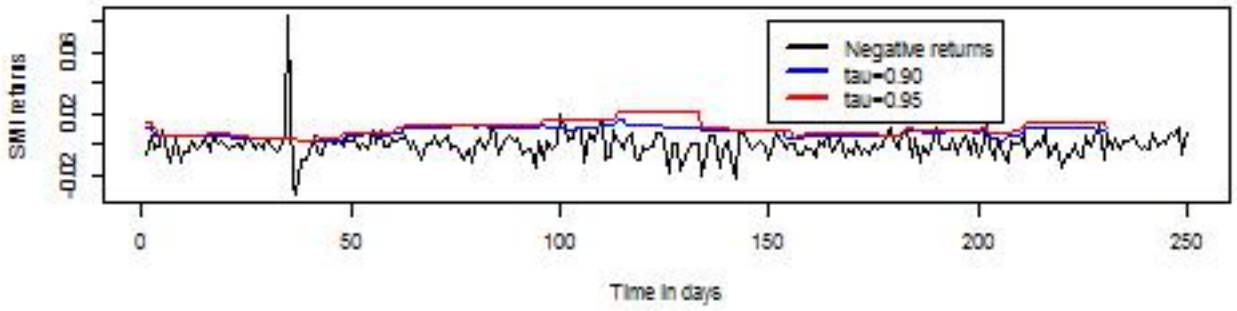
Results

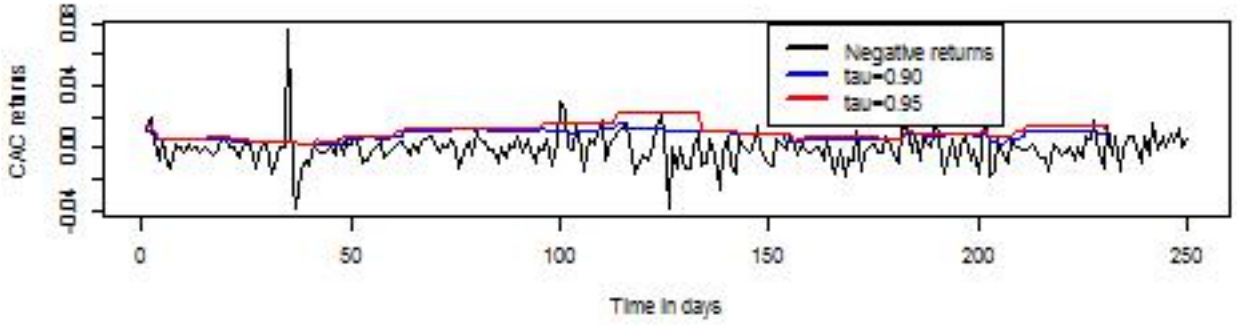


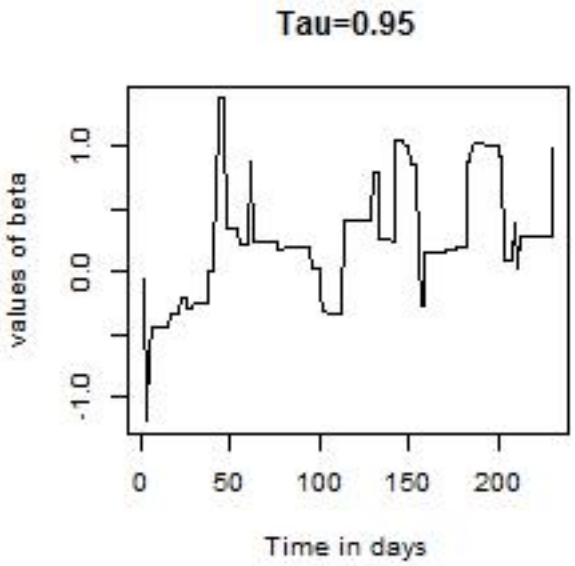


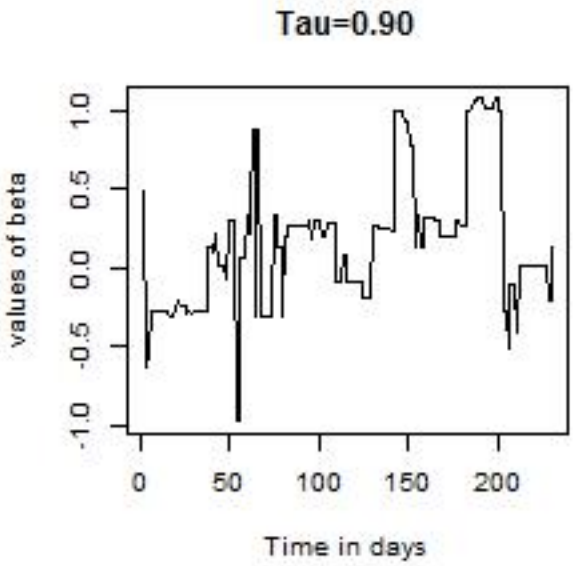
In the figure above, we can see that at low probabilities, values of beta have extreme negative values but as the value of probability increases, then there is remarkable increase in the value of Beta.

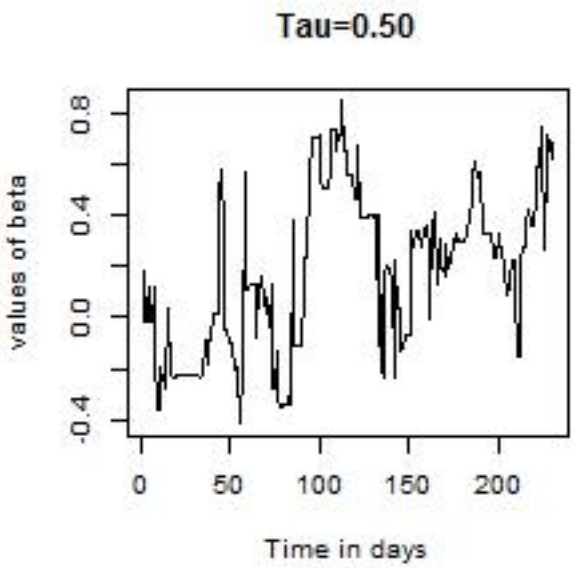

DAX returns
-0.05
0.00
0.05
0.10
Negative returns
tau=0.90
tau=0.95
0
50
100
150
200
250
Time in days

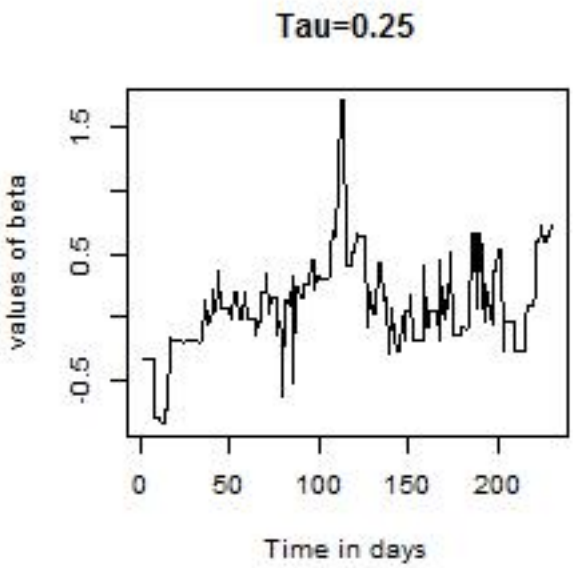

SMI returns
-0.02
0.02
0.06
Negative returns
tau=0.90
tau=0.95
0
50
100
150
200
250
Time in days

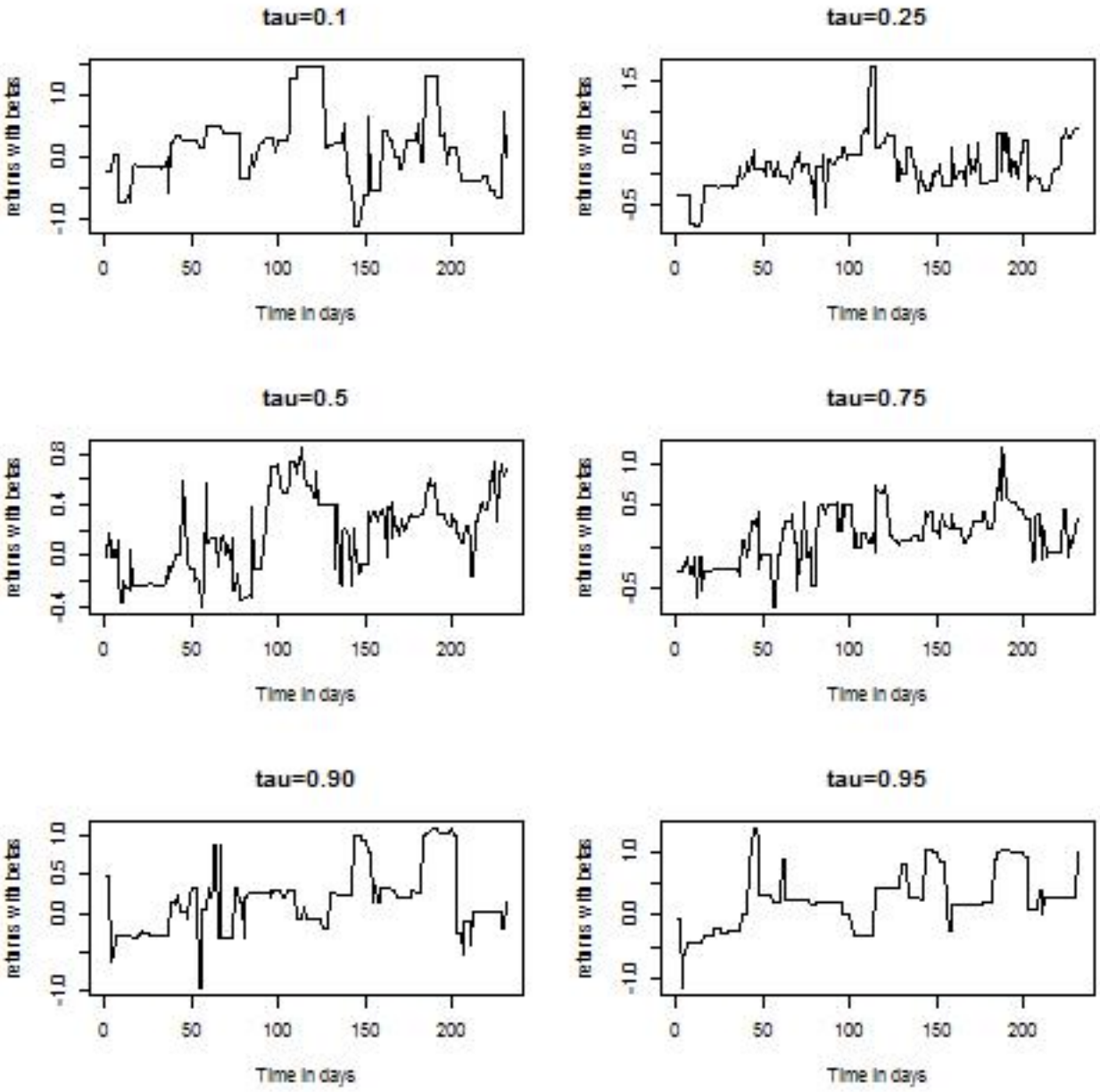

CAC returns
-0.04
0.00
0.04
0.08
Negative returns
tau=0.90
tau=0.95
0
50
100
150
200
250
Time in days

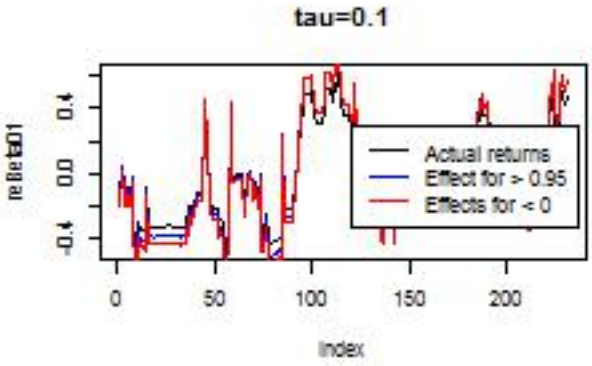

Tau=0.95
values of beta
-1.0
0.0
1.0
0
50
100
150
200
Time in days

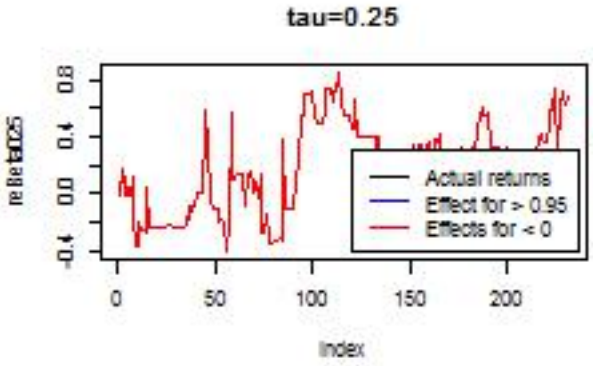

Tau=0.90
values of beta
-1.0
-0.5
0.0
0.5
1.0
0
50
100
150
200
Time in days

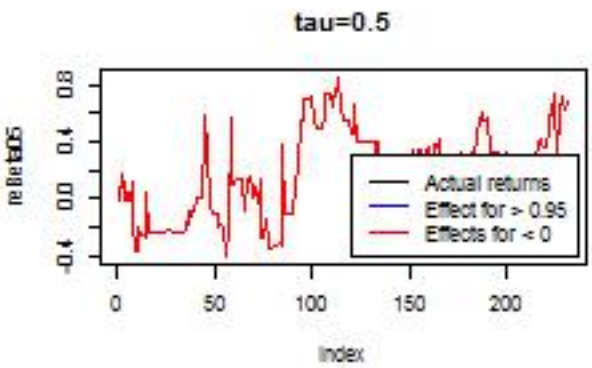

Tau=0.50
values of beta
-0.4
0.0
0.4
0.8
0
50
100
150
200
Time in days

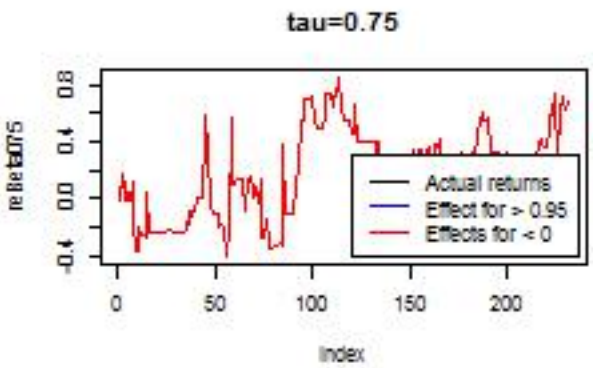

Tau=0.25
values of beta
-0.5
0.5
1.5
0
50
100
150
200
Time in days

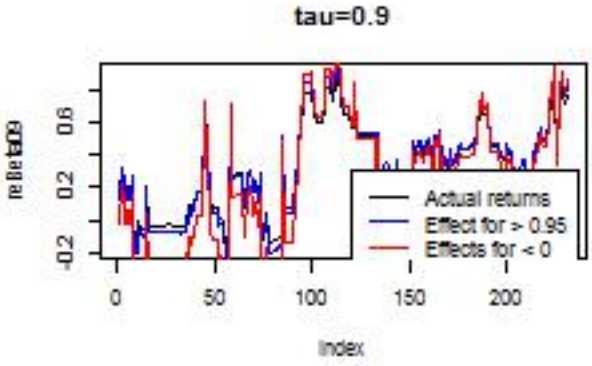

tau=0.1
returns with betas
-1.0
0.0
1.0
0
50
100
150
200
Time in days
tau=0.25
returns with betas
-0.5
0.5
1.5
0
50
100
150
200
Time in days
tau=0.5
returns with betas
-0.4
0.0
0.4
0.8
0
50
100
150
200
Time in days
tau=0.75
returns with betas
-0.5
0.5
1.0
0
50
100
150
200
Time in days
tau=0.90
returns with betas
-1.0
0.0
0.5
1.0
0
50
100
150
200
Time in days
tau=0.95
returns with betas
-1.0
0.0
1.0
0
50
100
150
200
Time in days

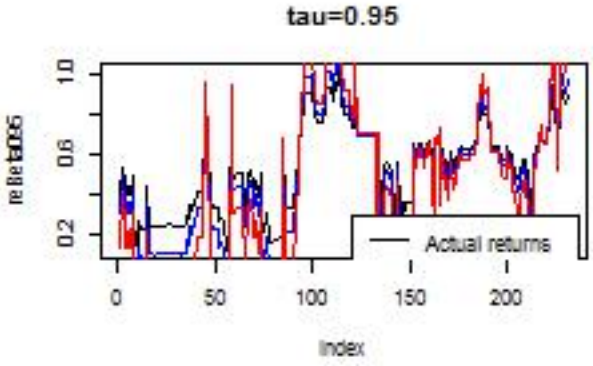

tau=0.1
Actual returns
Effect for > 0.95
Effects for < 0
Index


tau=0.25
Actual returns
Effect for > 0.95
Effects for < 0
Index


tau=0.5
Actual returns
Effect for > 0.95
Effects for < 0
Index


tau=0.75
Actual returns
Effect for > 0.95
Effects for < 0
Index


tau=0.9
Actual returns
Effect for > 0.95
Effects for < 0
Index


tau=0.95
Actual returns
Index

Conclusion and Recommendations

The Autoregressive coefficients follow a decreasing pattern over the quantiles of conditional return distribution. Negative returns show strong influence across the whole distribution than positive returns. Large negative returns influence greatly the pattern of coefficient estimates. Patterns at central quantiles are easy to predict compared to extremes of the distribution. We intent to explore the asymptotic properties of the extended conditional CAPM model through Monte Carlo Techniques. Also we itent to develop numerical results through re-sampling techniques.

References

[1] Black F., Jensen M. C. and Scholes M., (1972) *The capital asset pricing model: some empirical test, Studies in the Theory of Capital Market*, Praeger, New York.

[2] Breen W., Glosten L., and Jagannathan R., (1989), *Economic significance of Predictable Variations in Stock Index Returns*, Journal of Finance, **44**, pp. 1177-1189.

[3] Bickel P. J., and Lehmann E., (1976), *Descriptive statistics for nonparametric models. III: Dispersion*, Annals of Statistics, **4**, pp. 1139-1158.

[4] Bollerslev T., Engle R., and Wooldridge J., (1988), *A Capital-Asset Pricing Model with Time-Varying Covariances*, Journal of Political Economy, **96**, pp. 116-131.

[5] Chen N. F., (1991), *Financial investment Opportunities and the Macroeconomy*, The Journal of Finance, **46**, pp. 529-554.

[6] Chen Z., and Knez P. J., (1996), *Portfolio performance measurement: Theory and applications*, Review of Financial Studies, **11**, pp. 111-142.

[7] Engle R., Lilien D., and Robins R., (1987), *Estimating Time-Varying Risk Premia in the Term Structure: the ARCH-M Model*. Econometrica, **55**, pp. 391-407.

[8] Engle R., and Manganelli S., (2001) *CAViaR: Conditional autoregressive value at risk by regression quantiles*, preprint.

[9] Farma E. F., and French K. R., (1989), *Business conditions and expected returns on stocks and bonds*, Journal of Financial Economics, **25**, pp. 23-49.

[10] Ferson W. and Harvey C. R., (1991), *The Variation of Economic Risk Premiums*, Journal of Political Economy, **99**, pp. 385-415.

[11] Keim D. B., and Stambaugh R. F., (1986), *Predicting returns in the Stock and Bond Markets*, Journal of Financial Economics, **17**, pp. 357-390.

[12] Koenker R., and Bassett G. W., (1978) *Regression quantiles*, Econometrica, **46**, pp. 33-50.

[13] Koenker R., and Zhao Q., (1996), *Conditional quantile estimation and inference for ARCH models*, Econometric Theory, **12**, 793-813.

[14] Koenker R., and Xiao, Z., (2002), *Inference on the quantile regression process*, Econometrica, **70**, pp. 1583-1612.
[15] Koenker R., and Xiao Z., (2006), *Quantile Autoregression*, Journal of American Statistical Assiociation, **101**, pp. 980-990.
[16] Lintner J., (1965), *The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets*, Review of Economics and Statistics, **47**, pp. 13-37.
[17] Markowitz H., (1952), *Portfolio Selection*,The Journal of Finance, **7**, No. 1. pp. 77-91.
[18] Mossin J., (1966), *Equilibrium in a capital asset market*, Econometrica, **34**, pp. 768-783.
[19] Mwita P., (2010), *Estimation of T-period's ahead extreme quantile autoregression function*, African Journal of Mathematics and Computer Science Research, 3(**4**), pp. 60-70.
[20] Sharpe W., (1964), *Capital assets prices: a theory of market equilibrium under conditions of risk*, Journal of Finance, **19**, pp. 425-442.
[21] Sukono, Subanar and Dedi R., (2009), *Mean-VaR Portfolio Optimization under CAPM with lagged non constant Volatility and the Long Memory effect*, Journal of Quantitative Methods, **5**.
[22] Taylor, S., (1986) *Modeling Financial Time Series*. New York: Willey.
[23] Tobin J., (1958), *Liquidity preference as behaviour towards risk*, Review of Economics Studies, **7**, pp. 65-86.
[24] Xiao Z., and Koenker R., (2009), *Conditional Quantile Estimation for GARCH Models*, Boston College Working Papers in Economics, **725**.
[25] Wang K., (2002), *Nonparametric test of conditional mean-variance efficiency of a benchmark portfolio*, Journal of Empirical Finance, **9**, 133-169.
[26] Wang K., (2003), *Asset pricing with conditional information: A new test*. Journal of Finance, **58**, 161-196.