Analyzing Expected Returns of a Stock Using The Markov Chain Model and the Capital Asset Pricing Model

Ibrahim Adesokan[1]

Pan African University, Institute for Basic Science
Technology and Innovation, Nairobi Kenya

Philip Ngare

University of Nairobi, Nairobi, Kenya

Abdulhakeem Kilishi

University of Ilorin, Ilorin, Nigeria

Abstract

The chances of success of an investor in the stock market depends heavily on the decisions he takes based on his knowledge of the behavior of the stock market. In this study, the behavior of a stock on the Nigerian stock exchange market was studied. The Markov Chain with a threshold to determine movement between states, was used to estimate expected long and short-run returns, and the result was compared to the expected return of the Capital Asset Pricing Model. It was observed that the mean return of the stock and the expected return of the Capital Asset Pricing Model will be realized in the long-run regardless of the present state. The study indicates a way to forestall the problem of overpricing or under-pricing returns when using the Capital Asset Pricing Model.

---

[1]Corresponding author

Keywords: Stock market, Markov Chain, Capital Asset Pricing Model, Expected return, Limiting distribution, Steady-state probability

1 Introduction

A stock exchange is a well grounded legal framework which facilitates the trading of shares of companies or organizations in a country (Davou et al. 2013). A stock market which is on the rise is a good indicator of the country's economic strengths. An increase in share price is associated with increase in investments and a decrease in share price is linked with decrease in investments.

The dream of every person investing in the stock market is to make profit. However, the stock market is a financial market with a high volatility, so the success or failure of the investor heavily depends on the decisions taken. This depends on his knowledge of the stock market and strategies or models to predict the movement of prices that could arise as a result of many varying factors.

Over the years, many models like the Capital Asset Pricing Model (CAPM), the Geometric Brownian Motion(GBM),Markov chain model approach used by Zhang et al. (2009) to study and forecast the China's stock market trend etc, have been used to try to predict the behavior of the movement of stock prices, so as to reduce if not eliminate the risk of making losses in the stock market. However there is still a lot of debate as to which model is the most reliable. For example the GBM according to Abidin and Jaffa (2014) can be used to forecast a maximum of two weeks prices, even though their research was carried out on small companies. Also, the GBM fails to account for periods of constant values (Gajda and Wylomanska 2012). The CAPM uses a risk-free rate in determining expected return, but this risk free rate is also susceptible to volatility. All of these shortfalls give fuel to the debate as to which model is the most reliable for making decisions in the stock market. Hence there is a continuous need to come up with new models or review and upgrade existing models to try as much as is possible to reduce to the barest minimum the chances of failure in investing in a stock market.

The importance of making well informed decisions that would enhance the chances of success in the stock market can not be over-emphasized, one needs to observe the trend and behavior of an equity before purchasing stakes in it, as the size of loss which may arise from poor decisions cannot be overlooked. Since the stock market is a volatile market which has the random walk property, models which capture volatility would be expected to inform good predictions. Aguilera et al. (1999) noticed that daily stock price records do not conform to usual requirements of constant variance in the common statistical time series. The conventional CAPM which is used to estimate the expected return on an

asset was used by Prince and Evans (2013) to predict the expected returns for some stocks on the Ghana stock exchange, it was discovered that the model either under-valued or over-valued the asset as against the actual returns. Both cases are a challenge, since an investor might hold of from an asset due to the expectation of the CAPM while it was under-valued and vice-versa.

Zhang and Zhang (2009) introduced the concept of using a Markov chain model to predict stock market trend in the Chinese stock market. However the Markov chain unlike the CAPM only gives probabilities and not actual prices of stock and might not be sufficient on its own as an indicator.

This work uses the concept of a threshold to determine when a stocks return can be said to have risen, fallen, or remained static, which would lead to a change in the transition probabilities, thereafter analyzed the expected long and short-run return with the expected return of the conventional CAPM.

The specific objectives of this work are to determine a threshold in modeling movement of the stock returns, to calculate the expected long and short-run returns of the stock Using a three state Markov chain model, and to compare the expected long and short-run returns of the Markov chain model with the expected return of the Capital asset pricing model.

The second section of this article presents the models used, the third section focuses on the analysis of data and the results, and the final section presents discussion and conclusion of the study

2 Model Setup

This section generally presents discussion on the Markov chain model, the method of estimation of a fitting threshold, and the conventional Capital asset pricing model.

2.1 Markov chain model

The Markov chain model named after Andrey Markov, who produced the first theoretical results in 1906 (Eseoghene, J.I 2011). A generalization to countably infinite state spaces was produced by Kolmogorov in 1936.

For a process to be modeled by a Markov chain the present state of the process will depend only on the immediate past state, and transition probability matrices are the result of processes that are stationary in time or space; the transition probability does not change with time or space

Definition 2.1.1 A Markov chain is collection of random variables X_t (where the index runs through 0, 1, $\cdots$) having the property that, given the present, the future is conditionally independent of the past. In other words,

$$P(X_i = j | X_0 = i_0, X_1 = i_1, \cdots, X_{t-1} = i_{t-1}) = P(X_t = j | X_{t-1} i_{t-1}) \qquad (1)$$

2.2   Estimating Transition probabilities

The maximum likelihood method will be applied to estimate the transition probabilities
probabilities
Consider

$$P_{ij} = P_r(X_{t+1} = j | X_t = i) \tag{2}$$

Where $x_i \leq x_1, x_2, \ldots, x_n$ is a realization of the random variable $X_1{}^n$, and the probability of this realization is

$$P_r(X_1^n = x_1^n) = P_r(X_1 = x_1) \prod_{t=2}^{n} P_r(X_t = x_t | X_1^{t-1} = x_1^{t-1})$$

$$= P_r(X_1 = x_1) \prod_{t=2}^{n} P_r(X_t = x_t | X_1^{t-1} = x_1^{t-1}) \tag{3}$$

Which can be rewritten in terms of the transition probabilities P_{ij} to get the likelihood of a given transition matrix

$$L(p) = P_r(X_1 = x_1) \prod_{t=2}^{n} P_{(xt-1)(xt)} \tag{4}$$

Let $N_{ij} \equiv$ number of times i is followed by j in X_1^n and re-write the likelihood in terms of them

$$L(p) = P_r(X_1 = x_1) \prod_{i=1}^{k} \prod_{j=1}^{k} P_{ij}^{nij} \tag{5}$$

Maximizing the likelihood with respect to P_{ij}, the logarithm of (7) is first taken. Hence,

$$\iota(p) = logL(p) = logP_r(X_1 = x_1) + \sum_{i,j} n_{ij} logP_{ij} \tag{6}$$

$$\Rightarrow \frac{\partial \iota}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}}$$

Equating the first derivative of (8) with respect to P_{ij} to zero yields

$$\frac{n_{ij}}{P_{ij}} = 0$$

Considering $\sum_j P_{ij} = 1$ and picking one of the transition probabilities to express in terms of the others, say it's the probability of going to 1, so for each i, $P_{i1} = 1 - \sum_{j=2}^{n} P_{ij}$. Now, take derivatives of the likelihood and have $\frac{\partial}{\partial P_{i1}}$

$$\frac{\partial \iota}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}} - \frac{n_{i1}}{P_{i1}} \tag{7}$$

setting this equal to zero at the MLE $\hat{P}$,

$$\frac{n_{ij}}{\hat{P}_{ij}} = \frac{n_{i1}}{\hat{P}_{i1}} \tag{8}$$

$$\frac{n_{ij}}{n_{i1}} = \frac{\hat{P}_{ij}}{\hat{P}_{i1}} \tag{9}$$

since this holds $\forall j \neq 1$, we conclude $\hat{P}_{ij} \alpha n_{ij}$ and infact $\hat{P}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$.

2.3 Estimating Threshold

Consider the mean, median and mode as a means to establish the threshold, depending on what suits the data best.

Mean : This gives an estimate of the most common value, it is the only measure of central tendency whereby the deviation from the mean of each value is definitely zero. However using the mean to determine the threshold makes it vulnerable to influence of outliers.

Median : Unlike using the mean, this measure is less susceptible to the influence of outliers and skewed data. It is the mid-value of data-set arranged in order of magnitude.

Mode : This gives the most frequent value in a data-set. However, just like the mean, it is also susceptible to influence of outliers.

2.4 Estimating Expected Returns

The expected long and short-run returns were calculated using the formula given in KILI, (2013). the long-run is given by

$$\mu_R = \pi_j \mu_i \tag{10}$$

While the short-run is given by

$$\mu_r = P^n \mu_i \tag{11}$$

,

where μ_R and μ_r are the expected long and short-run returns respectively, π_j is the steady-state probability, P^n is the limiting probability and μ_i is the mean returns of state i.

2.5 Capital Asset Pricing Model

The CAPM is a simple yet powerful tool used by investors to determine the risk and reward of a stock. The model basically tells us how much compensation

is to be expected for taking risks.

The CAPM can be divided basically into two parts, the first part takes into consideration the time value of money, this is represented in the formula by the risk-free rate.

The second part of the CAPM formula considers risk and calculates the return an investor is required to take as compensation for taking on additional risk, since there's no reason to carry more risks if returns remain the same as or lower than the risk-free rate.

The CAPM's formula is given by

$$E_r = r_f + \beta(r_m - r_f) \tag{12}$$

where

E_r is the expected return on the stock

r_f is the risk-free rate

r_m is the expected market return and

β is the beta of the stock.

The β (beta) serves as a risk measure, it reflects how risky an asset is compared to overall risk of the market. So the higher the β of a stock, the riskier it is and hence the higher the return the investor expects. The relation $(r_m - r_f)$ which is the difference between the expected market return and the risk free rate is also referred to as the Market risk premium.

3 Data Analysis and Results

3.1 Data Analysis

The daily closing prices of Nestle Nigeria PLC from the Nigerian stock exchange was used in this study.

The daily returns were classified into three states depending on the threshold say "a". So the return can be classified as being higher than or lower than, or within "a" naira from the return of the previous day. Hence,

Fall State : If today's return is lower than the return of yesterday's by more than "a" naira.

Rise state : Here, today's return is greater by more than "a" naira than the return of yesterday.

Stable state: Here today's return is within or equal to "a" naira in relation to yesterday's return.

Let Y_n denote the closing day's price of the stock on the nth day, and Y_{n-1} denote the closing day's price of the previous day. The returns Z_n were

estimated using

$$Z_n = \frac{Y_n - Y_{n-1}}{Y_{n-1}} \tag{13}$$

The above states can be represented as a trinary random variable X_n denoted by

$$X_n = \begin{array}{ll} 1, & if\, Z_n > a \\ 2, & if\, |Z_n| \leq a \\ 3, & if\, Z_n < -a \end{array} \tag{14}$$

Hence we say the random variable X_n is a Markov chain of three states with state space $1, 2, 3$. Here "a" is the threshold value which will be gotten using appropriate measure of central tendency after observation of data. So we get a transition frequency of the form

State	Rise	Stable	Fall	Sum
Rise	n_{11}	n_{12}	n_{13}	S_1
Stable	n_{21}	n_{22}	n_{23}	S_2
Fall	n_{31}	n_{32}	n_{33}	S_3

where $n_{ij}(i, j = 1, 2, 3)$ represents the number of times transition is made from state i to state j, and $S_i(i = 1, 2, 3)$ represents the sum of values in each row i.

The transition matrix is obtained as shown in (9),

$$P_{ij} = \frac{n_{ij}}{\sum_j n_{ij}},$$

which is of the form

$$P = \begin{array}{c} \\ \end{array} \begin{pmatrix} \overset{Rise}{P_{11}} & \overset{Stable}{P_{12}} & \overset{Fall}{P_{13}} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix}$$

where P_{ij} represents the probability of moving from state i to state j.

For the CAPM, the beta which is a measure of the stock's volatility over time in relation to a market benchmark was estimated using the Microsoft excel software, as against using online calculators, this gives us full control and understanding of the stock. The β for the stock was estimated using the closing prices of the stock, and the chosen benchmark. The daily percentage change for the stock and the index, using the formula

$$D\%C = \frac{Y_n - Y_{n-1}}{Y_{n-1}} * 100 \tag{15}$$

where
Y_n denotes today's closing price, and
Y_{n-1} denotes yesterday's closing price.
then, we compute the β using the formula

$$\beta = \frac{Cov(stock's\%dailychange, index's\%dailychange)}{Var(index's\%dailychange)} \tag{16}$$

3.2 Results

The successive day's prices where used to calculate the returns Z_n. The mean of the returns Z_n denoted a was used as the threshold to determine movement between the states, where "a" $= 0.3639$ is the threshold value gotten. Also, μ the mean return of each state is given by $\mu = [3.1299, 2.6385, 3.3167]$. Below is the transition frequency.

State	Fall	Rise	Stable	Sum
Fall	17	17	24	58
Rise	24	28	16	68
Stable	18	23	70	111

The transition matrix is given by
P =

$$\begin{array}{ccc} Fall & Rise & Stable \\ \begin{pmatrix} 0.293103 & 0.293103 & 0.413794 \\ 0.352941 & 0.411765 & 0.235294 \\ 0.162162 & 0.207207 & 0.630631 \end{pmatrix} \end{array}$$

The transition matrix P is a probability vector which specifies the value of proportion for change in the stock's return movement in two consecutive days. From the transition matrix, the first row implies that if the return is in a fall state, the next day's return will fall or rise or be stable with the following percentages 29.31%, 29.31% and 41.38% respectively. The second row indicates that if the return today rises, the probability of it falling or rising or being stable are 35.29%, 41.18% and 23.53% respectively. The third row indicates that if the return today is stable with respect to yesterday's price, the probability of it falling or rising or being stable are given by 16.22%, 20.72% and 63.06% respectively. From the transition matrix P, it is realized that the chain is ergodic, since all of its states communicate and are aperiodic. Therefore the steady state probability vector is given by $\pi = [0.249706, 0.287465, 0.462829]$. This implies that 24.97% of the time, the return will move to a fall state,

28.75% of the time to a rise state, and 46.28% of the time to a stable state. The limiting probabilities of the daily returns were found to be

$$P^1 = \begin{pmatrix} \overset{Fall}{0.293103} & \overset{Rise}{0.293103} & \overset{Stable}{0.413794} \\ 0.352941 & 0.411765 & 0.235294 \\ 0.162162 & 0.207207 & 0.630631 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} \overset{Fall}{0.256461} & \overset{Rise}{0.292346} & \overset{Stable}{0.451194} \\ 0.286924 & 0.321768 & 0.391307 \\ 0.222945 & 0.263526 & 0.513529 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} \overset{Fall}{0.251521} & \overset{Rise}{0.289044} & \overset{Stable}{0.459435} \\ 0.261120 & 0.297681 & 0.441199 \\ 0.241638 & 0.280268 & 0.478094 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} \overset{Fall}{0.250245} & \overset{Rise}{0.287944} & \overset{Stable}{0.461811} \\ 0.253148 & 0.290536 & 0.456316 \\ 0.247278 & 0.285299 & 0.467423 \end{pmatrix}$$

$$\vdots$$

$$P^{17} = \begin{pmatrix} \overset{Fall}{0.249706} & \overset{Rise}{0.287465} & \overset{Stable}{0.462829} \\ 0.249706 & 0.287465 & 0.462829 \\ 0.249706 & 0.287465 & 0.462829 \end{pmatrix}$$

Figure 1 is the transition diagram representing transition between the three states. It clearly shows that each each state communicates with it self and all other states. Also the transition probabilities between the states are seen.
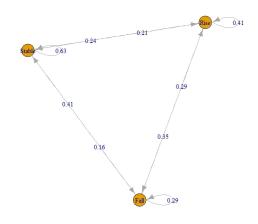
Figure 1: **Transition diagram**

Expected Returns

The expected long and short-run returns were calculated using the formula given in KILI (2013). The estimated expected long-run was gotten to be 0.001306117. The results for the expected short-run returns are presented in tables 1 and 2. The tables present the short-run expected returns for each state from the first day (t=1) to the seventeenth day (t=17) when the returns go into a steady state.

Table 1: **Expected Short-run returns**

State	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Fall	0.000110	0.001248	0.001299	0.001304	0.001306	0.001306	0.001306	0.001306	0.001306
Rise	0.0020197	0.001225	0.001272	0.001295	0.001303	0.001305	0.001306	0.001306	0.001306
Stable	0.001508	0.001387	0.001331	0.001314	0.001308	0.001307	0.001306	0.001306	0.001306

Table 2: **Expected Short-run returns**

State	t=10	t=11	t=12	t=13	t=14	t=15	t=16	t=17
Fall	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306
Rise	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306
Stable	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306	0.001306

The risk premium of the CAPM was gotten from the relation $(r_m - r_f)$ where r_m is the average return of the market for the year, estimated from daily closing prices of the Nigerian stock exchange All-share index. And r_f the average risk-free rate of return of the year, which is the yield on the government treasury bill which is relatively risk-free. Using equation (16), the β was found to be 0.1066, and using equation (12), the expected return was found to be 0.000336.

4   Discussion and Conclusion

This work presents an application of the Markov chain model and the Capital asset pricing model on same data set of daily returns of Nestle Nigeria PLC listed on the Nigerian stock exchange.

The result from the study shows that the overall average of returns of the stock (0.001302) will be realized in the long-run (i.e after 17 days regardless of present state). Also, depending on if the return is in a Fall, Rise, or Stable state, an investor can realize a return that is greater than the overall average return after four, five or two days respectively.

In addition, it was noticed that the expected return gotten from the CAPM will be realized in the short run after two days if the return is in the fall state, after one day if the return is in the rise state or stable state. Also regardless of present state of return the expected return of the CAPM will be realized in the long-run. This hints that the stock was not overpriced by the Capital asset pricing model. Hence it indicates that an investor will realize a positive return regardless of the present state in the long run.

In this study, the Markov chain model was used to estimate long and short-run expected returns. We find that regardless of present state of returns, the average returns will be realized after seventeen days. Also depending on the present state, the average return will be realized after a maximum of two days. The expected return from the CAPM was computed and compared to the long and short-run expected returns of the Markov chain model. The analysis suggests that the expected return of the CAPM is not over priced, so an investor will be better off investing on the stock. It is recommended that further research be carried out using a portfolio consisting of different stocks or on other instruments like gold and foreign exchange returns. Also since this work focuses on daily returns, more work can be done by analyzing smaller or larger interval of returns, like hourly, monthly or yearly returns of stock

References

[1] M.A. Aguilera, A.F. Ocaña, J.M. Valderrama, Stochastic modeling for evolution of stock prices by means of functional principal component analysis, *Applied Stochastic Models in Business and Industry*, **15** (1999), no. 4, 227-234. https://doi.org/10.1002/(sici)1526-4025(199910/12)15:4<227::aid-asmb388>3.0.co;2-c

[2] S. O. N. Agwuegbo, A. P. Adewole and A.N. Maduegbuna, A random walk model for stock market prices, *Journal of Mathematics and Statistics*, **6** (2010), no. 3, 342-346. https://doi.org/10.3844/jmssp.2010.342.346

[3] B. Aparna and Sarat CH. Kakaty, Analysis of Stock Market Price Behaviour: A Markov Chain Approach, *Int. J. Recent Sci. Res.*, **6** (2015), no. 10, 7061-7066.

[4] N.C. Davou, N.E. Samuel and T.K. Gokum, Markov Chain Model Application on Share Price Movement in Stock Market, *Computer Engineering and Intelligent Systems*, **4** (2013), no. 10, 1-10.

[5] E.J. Idolor, The Long Run Prospect of Stocks in the Nigerian Capital Market: a Markovian Analysis, *Journal of Research in National Development*, **9** (2011), no. 1, 388-400.

[6] Felix O. Mettle, Enoch N.B. Quaye, Ravenhill A. Laryea, A methodology for stochastic analysis of share prices as Markov chains with finite states, *Springerplus*, **3** (2014), 657. https://doi.org/10.1186/2193-1801-3-657

[7] S. B. Kiliç, Estimating Probability of Session Returns for Istanbul Stock Exchange 100 Index as Markov Chain Process, *Çukurova Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, **22** (2013), no. 1, 1-8.

[8] C. Okonta, O. Elem-Uche, C. Madu, A Markovian Analysis of the Nigerian Stock Market Weekly Index, *International Journal of Statistics and Applications*, **7** (2017), no. 2, 152-157.

[9] Oliver Ibe, *Markov Processes for Stochastic Modelling*, Elsevier, 2013.

[10] A. Prince, A. Evans, Does the Capital Assets Pricing Model (CAPM) Predict Stock Market Returns in Ghana? Evidence from Selected Stocks on the Ghana Stock Exchange, *Research Journal of Finance and Accounting*, **4** (2013), no. 9, 1-6.

[11] K. Reddy and V. Clinton, Simulating Stock Prices Using Geometric Brownian Motion: Evidence from Australian Companies, *Australasian Accounting, Business and Finance Journal,* **10** (2016), no. 3, 23-47. https://doi.org/10.14453/aabfj.v10i3.3