

Simulation of Traffic Congestion at Unsignalised Intersections using a Microscopic Traffic Flow Model

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Abstract

In this paper, we study and present a Microscopic Simulation of traffic in an unsignalised intersection using a Microscopic Car Following model based on the General Motors Theory (GM). The adverse effect caused by the traffic congestions is most notable in the largest cities, where traffic density is relatively high, with characteristically low and often variable speed. It extends the GM model by adding lane change (turning) manoeuvres which form the basis for describing the traffic flow at unsignalised intersections. We show how augmentation of traffic flow in most urban areas due to growth in transport and continual demand for it forms at those intersections. This is investigated through a mathematical simulation in C++ and analysed numerically. Microscopic car following model considers vehicles in discrete form and this discretization using the finite (forward) difference method. Vehicles are put in a simulation environment over a specified period of time over which they are considered for analysis.

Keywords: Microscopic Traffic; Car Following; General Motors; Simulation.

1. INTRODUCTION

The paper focuses on unsignalised intersections. Unsignalised intersections are examples of bottlenecks that are interesting in the study of traffic congestion since they are uncontrolled. Arguably, an efficient transport system is essential for the optimal functioning and prosperity of any modern economy. The quality of life, self-fulfilment, and personal freedom to some extent depend on mobility as a key catalyst to this satisfaction. To achieve mobility, this paper proposes a mathematical traffic simulation model using the General Motors microscopic car-following model to simulate traffic at these intersections. The proliferate development and the tremendous growth in the population of most cities has had a significant influence on the travel pattern(s) of the community from one place to another. This results from an increased transport demand which in most cases leads to a rise in the cost of living of the citizenry [17]. The transportation system is also affected by the annual increase of the vehicles on the road. For example, in 2014, the Bloomberg Business magazine reported that the number of cars on Nairobi's roads has doubled to 700,000 since 2012- infrastructure and traffic management remaining unchanged. Again, Bloomberg reported that there is a foreseeable increase in the number of these vehicles to about 9 million by 2050, (10). This naturally means there's are increases road congestion, especially during peak hours. Traffic congestion is a major problem of transport in many countries [3, 12] and it's not expected to end soon. This is always accelerated by the ever increasing urbanization of vast areas as urban extensions. In some urban centres, many roads were laid in an incremental manner to cater to the increased traffic demand [8,14]. The cities have developed in a disintegrated urban form spreading along major traffic corridors [2]. At the unsignalised intersections, traffic congestion may be caused by failure by drivers to adhere to traffic rules, aggressive drivers or timid drivers. Most of the major intersections are no longer able to cope with even the present traffic demand. The cause of this results from bottlenecks for example merging and intersections. The way vehicles enter their lane of choice without much regard to the other drivers makes congestion grow spreading far and wide. [18] describes microscopic (micro) simulation models as more and more widely used to support real-time control and management functions in the field of transportation planning, functional design, and operation engineering. Car following models predict the response of the following vehicle to the stimulus caused by the lead vehicle [18]. Improper understanding and consideration of all traffic scenarios/traffic characteristics at intersections leads to inaccurate interpretations of the results of the analysis which are key in making recommendations about the design and location of the intersection. A good understanding ensures that all the parameters and variables chosen for the study are well evaluated. Planning improvements and resource mobilization to expand the roads are quite expensive and unsustainable hence, cannot be relied upon. Therefore, it is very important to redevelop procedures for integrating various local traffic characteristics for a thorough analysis.

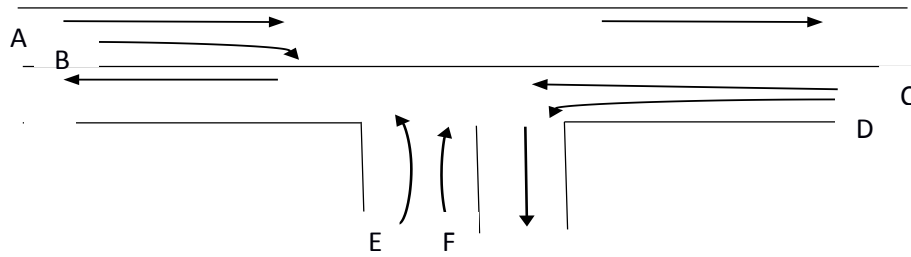


Figure 1: An Example of a T-Junction

2. MODEL PROBLEM

Our microscopic car-following model is developed in a classical procedure from the so-called ‘General Motors’ (GM) type car-following models used by [1], where we incorporate lane changes and merging abilities. The basic philosophy of car-following theories can be summarized as follows;

$[Response]_i \propto [Stimulus]$, for the i^{th} vehicle ($i = 1, 2, \dots$). Each driver in the traffic stream can respond to the surrounding traffic conditions by either accelerating or decelerating the vehicle. The stimulus may be composed of the speed of the vehicle, relative speeds, distance headway, etc. It is from this realisation that our model advances that the generalised General Motors Model car following It is given by the following equation(s).

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= C \left(\frac{v_{i+1} - v_i}{l_i - H} \right) + \frac{1}{T} (U(\rho_i) - v_i) \end{aligned} \tag{1}$$

where i is the vehicle, x_i is the position of the front bumper of vehicle i , \dot{x}_i is the velocity of the i^{th} vehicle, \dot{v}_i is the acceleration of the i^{th} vehicle, l_i is the spacing between two following vehicles, $U(\rho_i)$ is the equilibrium velocity, H is the length of vehicle i (considered as a constant here for all considered vehicles), T is the time vehicle i takes to reach the equilibrium velocity and C is a constant which scales the anticipation term.

Follow-the-Leader Model

The car following model proposed by General motors is based on follow-the-leader concept. This is based on two assumptions;

- (a) The more a vehicle moves at a high speed, the higher will be the spacing between the vehicles until it closes on the gap and
- (b) To avoid collision, a driver must maintain a safe distance with the vehicle ahead.

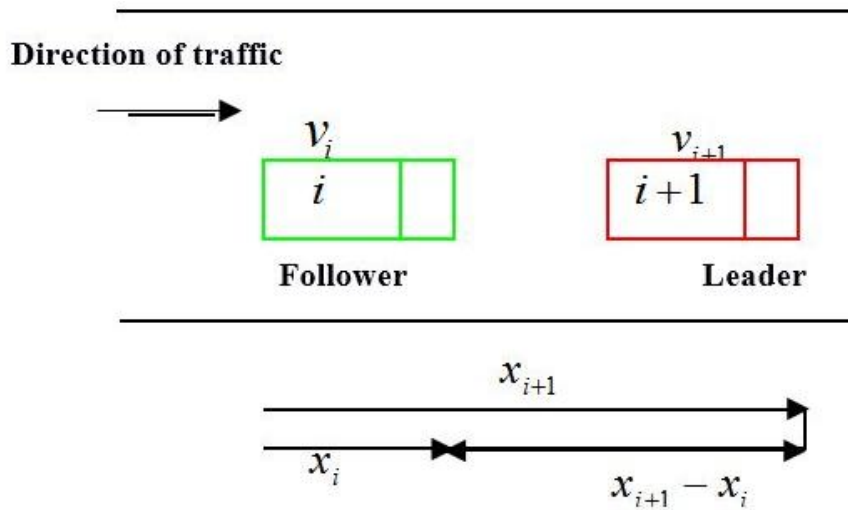


Figure 2: Notation for car-following model

The following vehicle is assumed to accelerate at time $t + \Delta T$, where ΔT is the interval of time required for a driver to react to a changing situation. We denote the location and speed of the vehicles at time $t \in R^+$ and the distance between the successive cars $l_i = x_{i+1} - x_i$ by $x_i(t)$ and $v_i(t)$ respectively and where $i = 1, \dots, N$. Considering (1), the local “density around vehicle i ” and its inverse (the local (normalized) “specific volume”) are respectively defined by; $\rho_i = \frac{H}{l_i}$ and $\tau_i = \frac{1}{\rho_i} = \frac{l_i}{H}$; Where ρ is the traffic density and H is the length of a car (vehicle).

A car-following model describes the motion of the following vehicles in response to the leading vehicle. Basically all vehicles that are not in free flow, adjust their driving behaviour to the vehicle ahead, to keep a safe headway. In theory, car following models can be used for all kinds of vehicles.

We describe the turning movement of along a T-Junction as shown in fig 3 below.

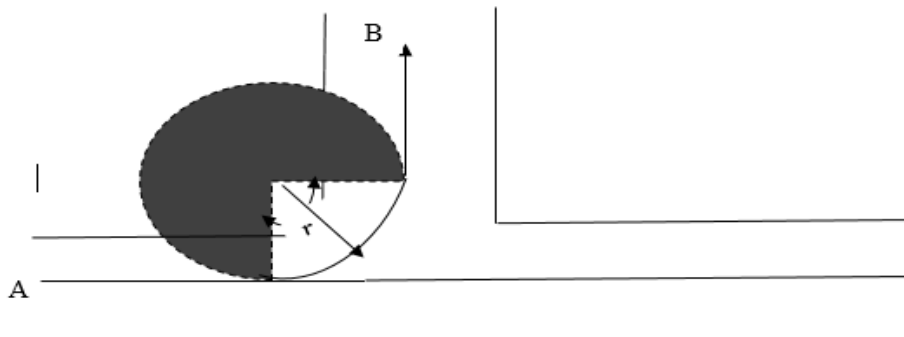


Figure 3: Movement along a T-Junction.

The initial condition for motion AB is given by $270 < \theta < 2\pi$. The initial condition will keep changing depending on the direction of the turn. For the motion described by AB, a vehicle moves from the straight path and to the connecting path. This motion makes an arc like path. A parametrization of the formed arc is done to help us in describing the motion. If we continue this path, the vehicle(s) moves in such a way that it forms part of a circle with a centre somewhere and with radius r which is always constant throughout the motion while θ as the vehicle moves along path AB. The movement is occurring on a two-dimensional plane on the x and y - axis where considering x and y in terms of the polar coordinates; $x = r \cos \theta$ and $y = r \sin \theta$. Looked this way, r is a vector and is given as $\vec{r}(\theta) = r(\cos \theta_i e_1 + \sin \theta_j e_2)$. A velocity v_i is applied on the vehicle as it travels through the desired distance xy or the r path.

Equations of the model.

Now we consider a dynamical system of n vehicles, translated as follows;

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n(n) \end{pmatrix}$$

The i^{th} component of the $x(t)$ is $x_i(t)$ $i \in \{1, \dots, n\}$ and $x(t)$ is again the vector position.

By differentiation, we obtain;

$$v(t, x) = \frac{dx}{dt} = \begin{pmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \frac{d}{dt} x_n(t) \end{pmatrix}$$

It can be seen that $\frac{dx}{dt} = f(t, x)$ where $f(t, x)$ is the vector field of the system. In our case, $f(t, x) = v(t, x)$.

By integration;

$$\int \frac{dx}{dt} dt = \int v(t, x) dt ,$$

For the turning motion, we now introduce an approximation for x which is given by;

$$x(t) = \int v(t, x) dt.$$

This approximation will give us a formula for the turning angle θ .

For each vehicle i , its position $x_i(t)$ is given by;

$$x_i(t) = \int v_i(t, x) dt; \text{ where } i = 1, 2, \dots, n$$

Looking at the Fig 2 below, there'll be two types of movements;

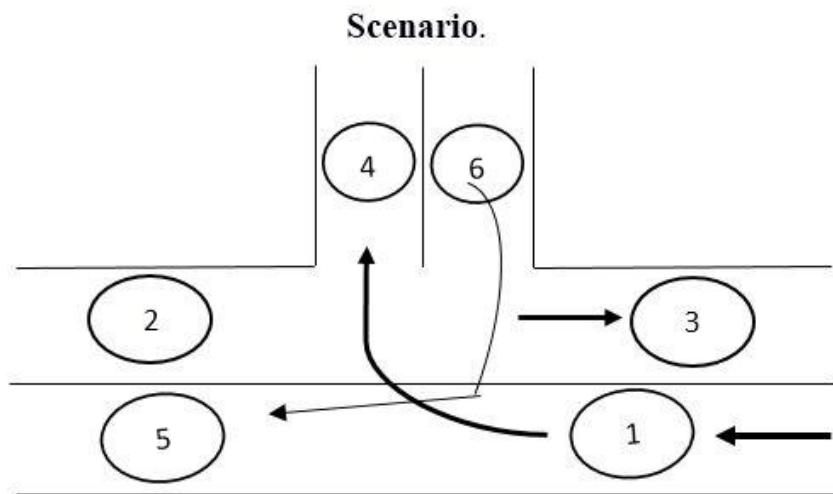


Figure 2: Interactions of vehicles at a T-Junction

- i) Straight Movements e.g. 1→5, 2→3.
- ii) Turning Movements e.g. 1→4, 6→5.

Described by;

Let l be the distance between the vehicles;

Let x_1 be the vehicle at position 1 intending to go to position 4

Let x_2 be the vehicle at position 2 moving straight to position 3

Now, at time instant t^* , vehicle v_1 can only get 4 *iff* there's a safe distance between itself and vehicle x_2 , that is;

$$x_1 - x_2 \geq l_{safe}(x_1, x_2, v_2)$$

Similarly, for vehicles at position 6 going to 5, applying the same conditions as described above; it has to check vehicles x_1 and x_2 this motion is described as below;

$$x_6 - x_2 \geq l_{safe}(x_6, x_2, v_2) \text{ and } x_6 - x_1 \geq l_{safe}(x_6, x_1, v_1)$$

The vehicle velocity in both motions is described by (2) and (3) respectively.

Then for every $i \in \{1, \dots, n\}$

$$\dot{x}_i = v_i(t) \quad (2)$$

$$\dot{x}_i(t) = r[\cos \theta_i(t)e_1 + \sin \theta_i(t)e_2] \quad (3)$$

Where;

$\theta_i(t) \cong \int v_i(t)dt$ and e_1 and e_2 are unit orthogonal vectors.

In both cases, the acceleration is described by;

$$\dot{v}_i = C \left[\frac{v(t)_{i+1} - v(t)_i}{l_i - H} \right] + \frac{1}{T} (U(\rho_i) - v_i), \text{ for } i \in \{1, \dots, n\} \quad (4)$$

$$\dot{v}_i = \frac{C}{H} \left[\frac{v_{i+1} - v_i}{\tau_i - 1} \right] + \frac{1}{T} (U(\rho_i) - v_i) \quad (5)$$

Doing simple transformations on (7) we get;

$$\dot{l}_i = v_{i+1} - v_i \text{ or } \dot{\tau}_i = \frac{1}{H} (v_{i+1} - v_i), \text{ for } i \in \{1, \dots, n-1\}$$

Discretizing using the Forward Finite Difference method.

In microscopic traffic modelling, each vehicle is considered on its own (as a single entity). Therefore, a discretization is done to get the equations that represent of each vehicles. We do this by the use of the finite forward finite difference method.

For any $i \in \{1, \dots, n-1\}$,

$$\frac{dx_i}{dt} = v_i$$

and

$$\dot{x}_i = v_i$$

Now;

Letting k be the time index and i the location index we have;

$$\frac{dx_i}{dt} = \frac{x_{i+1}^{k+1} - x_i^k}{\Delta t}$$

$$\frac{x_{i+1}^{k+1} - x_i^k}{\Delta t} = v_i$$

Where x_{i+1}^{k+1} is the position of the front bumper of the leading vehicle and x_i^k is the position of the front bumper of the following vehicle

Starting with n as zero we have;

$$\frac{x_i^1 - x_i^0}{\Delta t} = v_i$$

For the acceleration function;

Let k be the time index;

Then we have;

$$\frac{dv_i}{\Delta t} = \frac{v_i^{k+1} - v_i^k}{\Delta t}$$

Where v_{i+1}^{k+1} is the velocity of the leading vehicle and v_i^k is the velocity of the following vehicle.

Then we get:

$$\frac{v_{i+1}^{k+1} - v_i^k}{\Delta t} = C \frac{(v_{i+1} - v_i)}{l_i - H} + \frac{1}{T} (U(\rho_i) - v_i)$$

Existence and Uniqueness of Results

It is very important for us to check if the results exist and if yes, if they are unique. This is done by invoking the Cauchy Lipschitz Theorem.

Statement of the Cauchy Lipschitz Theorem

Let $U \subset R^n$ be an open set and $f : U \times [0, T] \rightarrow R^n$ a continuous function which satisfies the Lipschitz condition $|f(x_1, t) - f(x_2, t)| \leq M|x_1 - x_2| \forall (x_1, t), (x_2, t) \in U \times [0, T]$ (where M is a given constant). Let us consider the initial value problem described by $\begin{cases} \dot{x} = f(x, t) \\ x(0) = x_0 \end{cases}$ where $x \in U$ and $t \in [0, T]$. If $x_0 \in U$ then for some positive, δ there is a unique solution $x : [0, \delta] \rightarrow U$ of the initial value problem.

We take the hypothesis where our model equations (4), (5) and (6) are lipschitzian.

From the above sets of ODEs, we have;

$$\dot{x}_i = v_i(t) \tag{6}$$

$$\dot{x}_i = r(\cos \theta_i(t)e_1 + \sin \theta_i(t)e_2) \tag{7}$$

$$\dot{v}_i = C \frac{(v_{i+1}(t) - v_i(t))}{l_i - H} + \frac{1}{T} (U(\rho_i) - v_i) \tag{8}$$

$$\dot{Z} = F(Z) \text{ where } Z = \begin{pmatrix} x_i \\ \vdots \\ x_n \\ v_i \\ \vdots \\ v_n \end{pmatrix}, \quad \dot{Z} = \begin{pmatrix} \dot{x}_i \\ \vdots \\ \dot{x}_n \\ \dot{v}_i \\ \vdots \\ \dot{v}_n \end{pmatrix} \text{ and also;}$$

$$F(Z) = \begin{pmatrix} v_1 \\ \vdots \\ \vdots \\ \vdots \\ v_n \\ C\left(\frac{v_2(t) - v_1(t)}{l_i - H} + \frac{1}{T}(U(\rho_1) - v_1)\right) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C\left(\frac{v_{n+1}(t) - v_n(t)}{l_n - H} + \frac{1}{T}(U(\rho_n) - v_n)\right) \end{pmatrix}$$

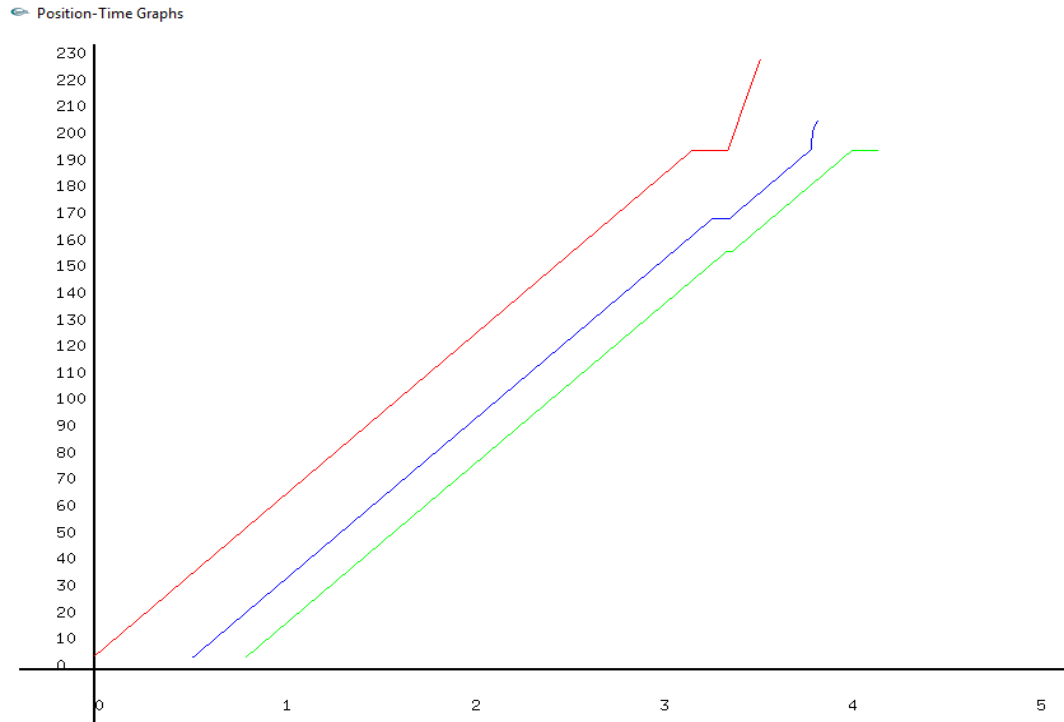
$$\dot{Z} = G(Z) \text{ where } Z \text{ is the same as above and } G(Z) = \begin{pmatrix} r \cos \theta_1 e_1 + r \sin \theta_1 e_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ r \cos \theta_n e_1 + r \sin \theta_n e_2 \\ C \frac{(v_2(t) - v_1(t))}{l_1 - H} + \frac{1}{T} (U(\rho_1) - v_1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C \frac{(v_{n+1}(t) - v_n(t))}{l_n - H} + \frac{1}{T} (U(\rho_n) - v_n) \end{pmatrix}$$

Taking the hypotheses as in the Cauchy Lipschitz theorem, we have $\dot{x}(t) = v(t)$ and $\dot{x}(t)$ is a continuous vector function and also $\dot{v}(t)$ is a Lipschitz continuous vector function. Then $x(t)$ and $v(t)$ are continuously differentiable. The solution of each system of ordinary differential equation is called the integral curve.

RESULTS AND DISCUSSION

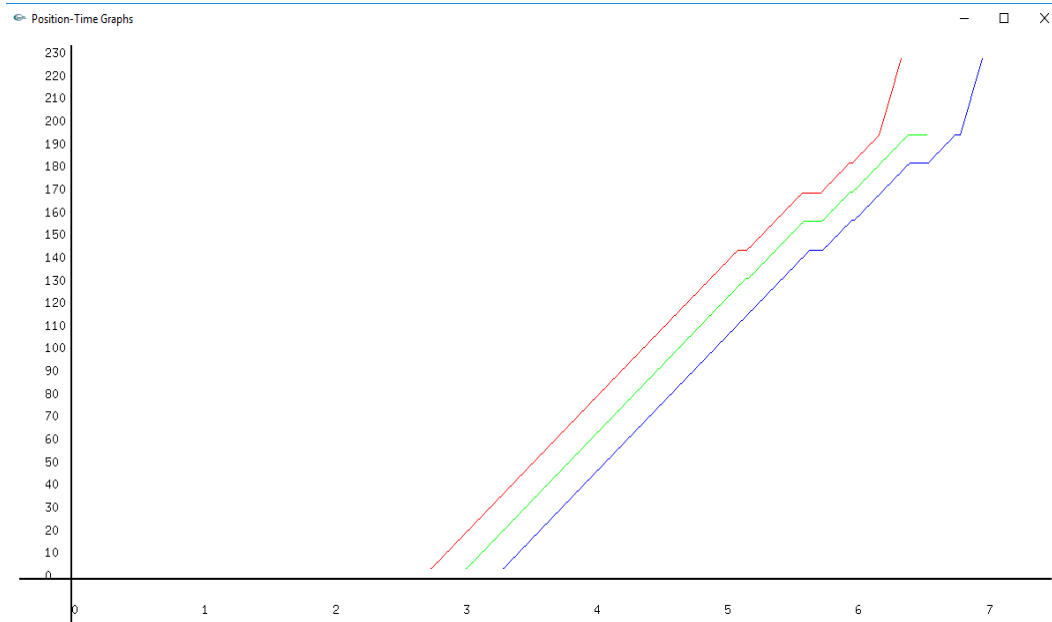
The results presented here are an outcome of the implementation of the finite difference method scheme in C++ programming language (both code and plots have been produced in C++). The vehicles are randomly generated. A hierarchical description of the vehicles id used in implementing their movement. The hierarchy specifies their claim on the right-of-way at the common intersecting space. In general, first in the hierarchy is the through movement on the Major Street; second is the right turn from a Major Street. For example, if in a situation there is a vehicle on the right turn movement and another on the conflicting through movement, the latter uses the intersection first while the former waits until the latter clears the intersection. If some other movement is still lower down the hierarchy (like the right turn from Minor Street), then a vehicle for that movement has to wait until the vehicles on the movements higher up in the hierarchy has cleared the intersection. The Red, the Blue and the Green Trajectories represent the preceding, the following and the turning vehicles respectively.

Position - Time Graph for First 3 vehicles in the Simulation



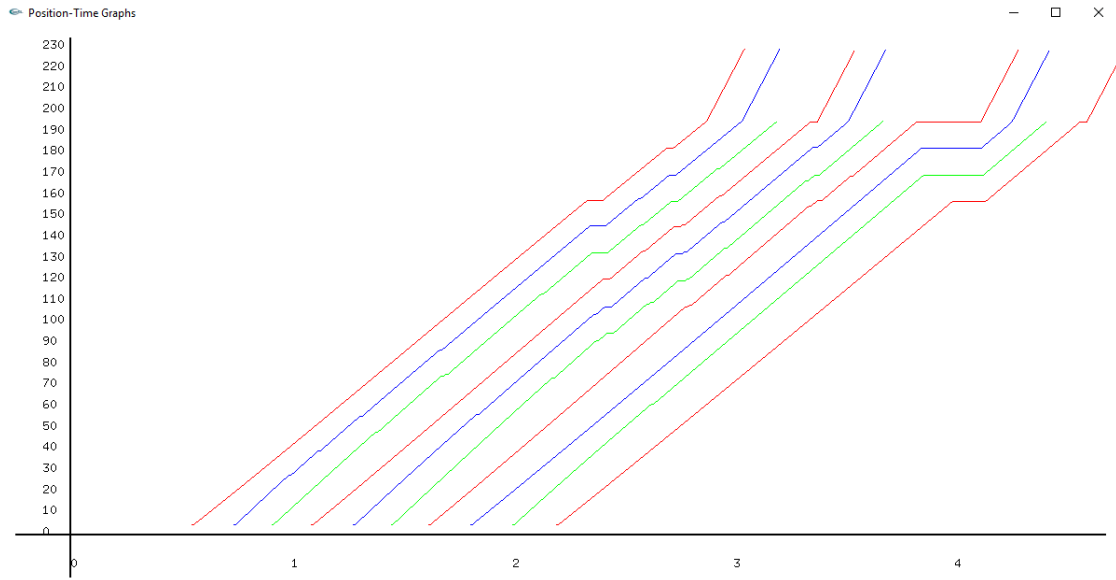
The preceding vehicle changes position uniformly with respect to time and its unchanging velocity. When it reaches the junction, the vehicle momentarily waits for another vehicle to turn after which it proceeds to drive past the junction at a higher speed hence the sharp gradient and the vehicle exists the junction. The following vehicle (blue vehicle) accelerates using the General Motors Car following model equations therefore the vehicle is solely dependent on the preceding vehicle (red). This is seen at the point where the following vehicle takes the preceding vehicle's behaviour by stopping and moving when it starts moving. The Turning vehicle's behaviour is as well depicted by the characteristics of the behaviour of the second vehicle. As seen in the Graph, the vehicle stops for some time and resumes motions as soon as the leading vehicle (Second) resumes movement. This clearly shows that the behaviour of the first vehicle was back-chained to all the following vehicles via the follow the leader behaviour as described by the General Motors Car Following Model.

Position - Time Graph for the 11th, 12th and 13th Vehicles in the Simulation



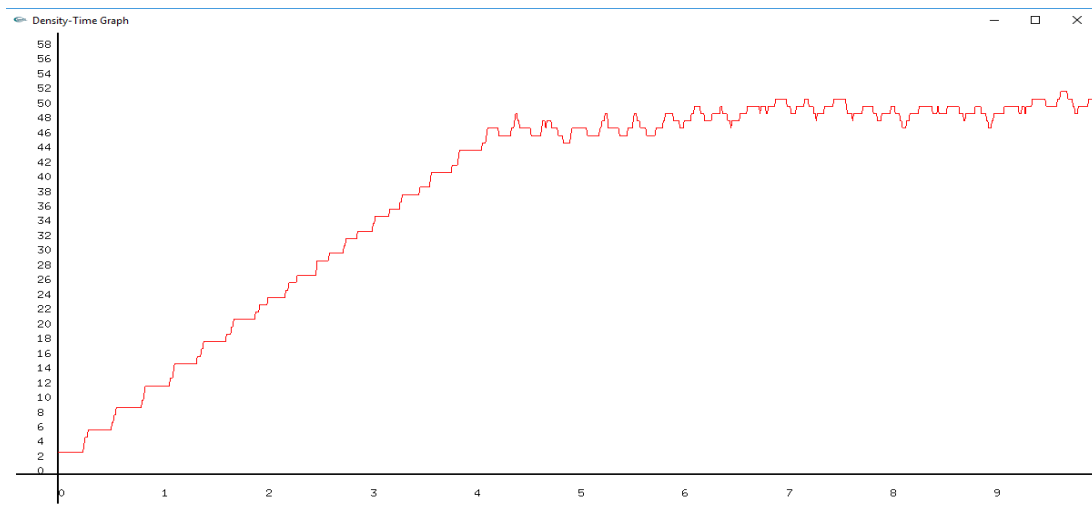
This is a case of 3 consecutive vehicles. The front vehicle is a consequence of the first vehicle having followed another vehicle (vehicle 10). We see that, after the vehicle drove for two seconds, it seized moving and then drove for some another a second stopped again, then moved again, stopped for a while and then proceeded to the junction after which it went straight almost immediately after it reached the junction. The second vehicle replicates the behaviour until it turns at the junction. Contrary, the 13th vehicle, does not immediately replicate the behaviour of the preceding vehicle (12th) although it accelerates with respect to the behaviour of the preceding vehicle. This shows that the vehicle doesn't blindly follow the preceding vehicle i.e. it first checks if the gap between itself and the preceding vehicle is less than the safe distance before it stops. When the 12th vehicle stops for the 2nd time, the 13th vehicle stops too and proceeds to according the behaviour of the preceding vehicle but as when it reaches the junction, it also stops momentarily even though the preceding vehicle had already turned. This implies that the stopping was as result of presence of another turning vehicle hence it had to give way.

Position-Time Graph for 10 vehicles in the Simulation



This graph illustrates the exact sequence of propagation of the behaviour of the preceding vehicle especially in the case where jam has occurred. Since there is a high density of vehicles, when the preceding vehicle stops for some time, it causes five more vehicles to stop. The 7th vehicle is affected by the jam propagation when the condition gap is greater than safe distance becomes false. This is the same case for the 8th, 9th and 10th vehicle. We therefore conclude that in so long as the gap between the preceding and the following vehicles is greater than the safe distance, the vehicle will always accelerate.

Traffic Density-Time Graph for the Traffic Simulation



This graph illustrates how a jam grows around a T-Junction. In the graph, we see that before the simulation starts there are few vehicles on the road. As more vehicles are generated the density of the traffic on the road grows gradually over time. In some instances, the density remains constant when there's a balance between the incoming and outgoing vehicles. As more and more vehicles are generated, the density increases until all the lanes are jammed. This creates a Traffic Shock wave which propagates backwards. By the use of the Stop and Go Condition that is based on prioritization, the density fluctuates as vehicles leave and enter the junction respectively but the density is maintained at interval 45-55. This explicitly means that the jam will never end.

CONCLUSION

In this paper, we presented the turning logic in the General Motors Car Following Model. Also, the paper clearly depicts the formation of jams at unsignalised intersections. Vehicles with lower priorities to turn, are forced to wait until no higher ranked conflicting vehicle turns. The stopping is propagated to the following vehicles which leads to the gradual growth of traffic jam over time. Further, if the vehicles move at low speeds, jam development rate is higher and the contrary is always true. Finally, the paper establishes that for safe merging, several strategies must be employed; prioritization where vehicles moving straight and or turning right from the main road/lane into a minor road or lane are given the highest priority. Second in that order are those vehicle turning left from the main or turning right into the main road. The lowest priority is given to those turning left into the main road. Maintaining a Safe Distance between the preceding and the following vehicle is another very impressive strategy for safe merging. A vehicle should also leave the junction at a higher velocity if it enters so that it gives way to the others

REFERENCES

- [1] Borsche, R., Kimathi, M., & Klar, A. (2012). A class of Multiple-Phase Traffic Theories for Microscopic, Kinetic and Continuum Traffic Models.
- [2] CES. (2006). Vijayawada City Development Plan (Chapter 4.6 Traffic and Transportation). Project: *Comprehensive Traffic and Transportation Study for Vijayawada City*. Consulting engineering services Ltd., India (CES). Document: 2006042/TT/Rep-02/Chap-3
- [3] Chen, C., Wang, Y., Li, L., Hu, J., Zhang, Z. (2012). *The retrieval of intra-day trend and its influence on traffic prediction*. Transp Res Part C Emerg Technol 22:103–118
- [4] Delis, A. I., Nikolos, I. K., & Papageorgiou, M. (2014). High-resolution numerical relaxation approximations to second-order macroscopic traffic flow models. *Transportation Research Part C: Emerging Technologies*, 44, 318–349. doi:10.1016/j.trc.2014.04.004
- [5] Gazis, D. C., R. Herman and R. B. Pott. Car-following theory of steady-state traffic flow. *Operations Research*, Vol. 7, 1959, pp. 499-505.

- [6] Gazis, D. C., R. Herman and W. R. Richard. Nonlinear follow-the-leader models of traffic flow. *Operations Research*, Vol. 9, 1961, pp. 545-567.
- [7] Gipps, P. G. (1986). A model for the structure of lane changing decisions. *Transportation Research Part B*, 20B (5), (pp. 403-414).
- [8] Goerigk M, Schachtebeck M, Schöbel A (2013) Evaluating line concepts using travel times and robustness. *Public Transp* 5:267–284
- [9] Helbing, D., Hennecke, A., Shvetsov, V., Treiber, M. (2002). Micro-and macro-simulation of freeway traffic. *Mathematical and Computer Modelling*, 517 – 547.
- [10] [http:// www.bloomberg.com/news/articles/2014-03-25/nairobi-traffic-loses-570-000-a-day-asno-2-africa-hubmberg.com](http://www.bloomberg.com/news/articles/2014-03-25/nairobi-traffic-loses-570-000-a-day-asno-2-africa-hubmberg.com)
- [11] http://www.ops.fhwa.dot.gov/congestion_report/chapter4.htm
- [12] Huang, S., Sadek, A.W. (2009). *A novel forecasting approach inspired by human memory: the example of short-term traffic volume forecasting*. *Transp Res Part C Emerg Technol* 17:510 525
- [13] Koutsopoulos, H. N., & Yang, Q. (1996). A microscopic traffic simulator for evaluation of dynamic traffic management systems. *Transportation Research Part C Emerging Technologies*, 4(3), (pp. 113 -129).
- [14] Lo HK, Lam WHK, Wong SC, Leung JMY (2012) Advanced systems for public transport scheduling and network design. *Public Transp* 3:1–2
- [15] Ntousakis, A., Porfyri, K., Nikolos, K., & Papageorgiou, M. (2014). Assessing the Impact of a Cooperative Merging System on Highway Traffic Using a Microscopic Flow Simulator.
- [16] Pipes, L. A. An operational analysis of traffic dynamics. *Journal of Applied Physics*, Vol. 24, No. 3, 1953, pp. 274-281.
- [17] Ruskin, H. J. (2015, January 15). *Modelling Traffic Flow at an Urban Unsignalized Intersection*. Retrieved from *Modelling Traffic Flow at an Urban Unsignalized Intersection*:<http://www.uvm.edu/~pdodds/files/papers/others/2002/ruskin2002a.pdf>
- [18] Zhang, Y. (2004). *Scalability of car following and lane changing models in microscopic traffic simulation systems*. PhD Thesis Southeast University. p 157

