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Unsteady hydromagnetic flow between parallel plates both moving in the presence of a constant pressure gradient

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Abstract: ***An unsteady magneto hydrodynamic viscous incompressible electrically conducting fluid flow between two parallel porous plates of infinite length in x and z directions subjected to a constant pressure gradient in the presence of a uniform transverse magnetic field applied parallel to the y axis with the plate moving with a time dependent velocity is analyzed. Two cases where the plates are moving (i) in the same direction, (ii) in the opposite direction while fluid suction/injection takes place through the walls of the channels with a constant velocity for suction and injection has been investigated. The nonlinear partial differential equation governing the flow are solved numerically using the finite difference method and implemented in MATLAB. The results obtained are presented in graphs. The velocity profiles, the effect of pressure gradient, magnetic field, time and suction /injection on the flow and the effects of varying various parameters on the velocity profile are discussed. A change on the parameters is observed to either increase, decrease or to have no effect on the velocity profile.***

Index Terms— **Pressure gradient, Suction and Injection, Same velocity.**

I. INTRODUCTION

MHD flows are characterized by a basic phenomenon which is the tendency of magnetic field to suppress vorticity that is perpendicular to itself which is in opposite to the tendency of viscosity to promote vorticity. MHD Couette flow is studied by a number of researchers due its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering. The MHD flow between porous plates studied has many important applications in areas such as the designing of cooling systems with liquid metals, geothermal reservoirs, in petroleum and mineral industries, in underground energy transport, accelerators, MHD generators, pumps, flow meters, purification of crude oil, polymer technology and in controlling boundary layer flow over aircraft wings by injection or suction of fluid out of or into the wing among many other areas. Researchers have studied unsteady channel or duct flows of a viscous and incompressible fluid with or without magnetic field analyzing different aspects of the problem. Katagiri [1] investigated unsteady hydro magnetic Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel.

Muhuri [2] considered this fluid flow problem within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel. Soundalgekar [3] investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under transverse magnetic field. The effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel, studied by Muhuri [2]. The work by Muhuri [2] was later analyzed by Govindrajulu [4]. Mishra and Muduli [5] discussed effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion when one of the plates starts moving with a time dependent velocity. In the above mentioned investigations, magnetic field is fixed relative to the fluid. Singh and Kumar [6] studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when fluid flow within the channel is induced due to time dependent movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. Singh and Kumar [6] considered two particular cases of interest in their study viz. (i) impulsive movement of one of the plates of the channel and (ii) uniformly accelerated movement of one of the plates of the channel and concluded that the magnetic field tends to accelerate fluid velocity when there is impulsive movement of one of the plates of the channel and when there is uniformly accelerated movement of one of the plates of the channel. Katagiri [1] studied the problem when the flow was induced due to impulsive motion of one of the plates while Muhuri [2] studied the problem with accelerated motion of one of the plates. Both had considered that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar [6] considered the problem studied by Katagiri [1] and Muhuri [2] in a non-porous channel with the magnetic lines of force fixed relative to the moving plate. S. Ganesh, S.Krishnambal, [7] studied unsteady MHD stokes flow of a viscous fluid between two parallel porous plates. They considered the fluid being withdrawn through both walls of the channel at the same rate. Various aspect of the flow problems in porous channel have been studied, Bég *et al.* [8], studied unsteady magnetohydrodynamic Hartmann-Couette flow and heat transfer in a Darcian channel with Hall current, ionslip, viscous and Joule heating effects. Makinde *et al.* [9] studied unsteady hydromagnetic flow of a reactive variable viscosity third-grade fluid in a channel with convective cooling while Vieru *et al.* [10] studied the Axial Flow of Several Non-Newtonian Fluids through a Circular Cylinder.

Seth *et al.* [11], studied the problem considered by Singh and Kumar [6] when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest, viz (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate. Seth *et al.* [11] concluded that the suction exerted a retarding influence on the fluid velocity whereas injection has accelerating influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate. Ismail *et al.* [12]. MHD flow between two parallel plates through porous medium with one in uniform motion and the other plate at rest and uniform suction at the stationary plate. They used the Similarity transformation method to solve the problem and concluded that the axial velocity of the fluid decreases as density, time, and Hartmann number increases. The Axial velocity of the fluid increases as average entrance velocity increases Transverse velocity of fluid increases as density, Hartmann number and suction increases. Joseph *et al.* [13] studied Unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer with the lower plate considered porous. They concluded it shows that magnetic field has significant effect to the flow of an unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. Kiema *et al.* [14] considered laminar viscous incompressible fluid between two infinite parallel plates when the upper plate is moving with constant velocity and the lower plate is held stationary under the influence of inclined magnetic field and concluded that the increase in magnetic field strength and magnetic inclination results into decreases in the velocity profiles. Onyango *et al*. [15] considered magneto hydrodynamic flow between two parallel porous plates with injection and suction in the presence of a uniform transverse magnetic field with the magnetic field lines fixed relative to the moving plate with a constant pressure gradient and concluded that the magnetic field, pressure gradient, time and injection have an accelerating influence on the fluid flow with a constant pressure gradient in the direction of the flow on both cases of suction and injection while viscosity and suction exert a retarding influence. Extensive researches have been done on the flow between parallel plates. This study is with consideration when both plates are motion with the same velocity in the same direction and in opposite directions. This work presents findings of studies on MHD couette flow problem between porous plates with magnetic field lines fixed relative to the moving upper plate with suction and injection on the plates.

II. MATHEMATICAL FORMULATION

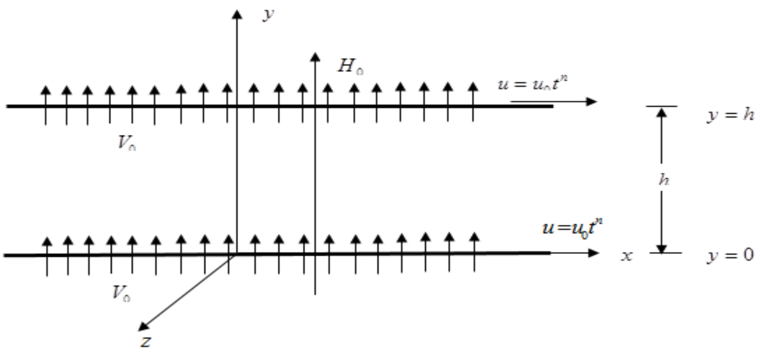

Fig1: Physical model of the problem

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This study considers the flow of unsteady viscous incompressible electrically conducting fluid between two parallel porous plates $y=0$ and $y=h$ of infinite length in x and z directions with a constant pressure gradient in the presence of a uniform transverse magnetic field H_o applied parallel to the y axis.

Initially (when time $t \leq 0$), the fluid and the porous plates of the channel are assumed to be at rest. When time $t>0$, the lower plate ($y=0$) and the upper plate ($y=h$) starts moving with time dependent velocity $u_o t^n$ (where u_o is a constant and n a positive integer) in the x direction with the fluid suction/injection takes place through the walls of the channel with uniform velocity V_o where $V_o>0$ for suction and $V_o<0$ for injection.

The velocity and the magnetic fields are given as $q=(u,v_0,0)$ and $\vec{H} \equiv (0,H_0,0)$ respectively.

The magnetic forces $=\sigma\mu_e{}^2 H_0 \times Velocity$

From the Navier Stokes equation

$$\rho\frac{\partial u}{\partial t}+\rho u\nabla u=-\nabla P+\mu\nabla^2 u+F \tag{1.0}$$

$$\rho\frac{\partial u}{\partial t}+\rho u\nabla u=-\nabla P+\mu\nabla^2 u+J\times B \tag{1.1}$$

The flow is incompressible (the density ρ, is considered a constant) and is considered in one dimension along the x- axis hence the Navier stokes equation along the x-axis is given as

$$\rho\frac{\partial u}{\partial t}+\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+J\times B \tag{1.2}$$

For a Couette flow $-\frac{\partial P}{\partial x}=0$ but for the analysis $-\frac{\partial P}{\partial x}$ = a constant β^*. The two plates are infinite in length hence $\frac{\partial u}{\partial x}=0$.The fluid is injected on the lower plate with a constant velocity V_0 and is also sucked from the upper plate at the same constant velocity V_0.The general equation governing the flow reduces to

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\beta^*}{\rho}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{(-\sigma\mu_e{}^2 H_0{}^2 u)}{\rho} \tag{1.3}$$

Where $\beta=\frac{\beta^*}{\rho}$, and

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2 H_0{}^2 u}{\rho} \tag{1.4}$$

where $\upsilon=\frac{\mu}{\rho}$

The magnetic field lines are fixed relative the moving plates (The upper plate and the lower are accelerating uniformly–a function of time) hence the velocity is considered as a relative velocity and reflects how fast the fluid is moving relative to the moving plates. The general equation governing the flow

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2 H_0{}^2\left(u-u_0 t^n\right)}{\rho} \tag{1.5}$$

For consideration of the two cases of interest viz. (i) movement of the plates in the same direction (i.e. n = 1) and (ii) movement of the plates in the opposite direction (i.e. n = 1).

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Case I. Movement of the plates in the same direction (i.e. *n* = 1)

Taking $n=1$, for a case of uniform acceleration, the governing equation for the flow becomes

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e{}^2H_0{}^2\left(u-u_0t\right)}{\rho} \tag{1.6}$$

With the boundary conditions defined as;

$$\begin{aligned} &u=0 && 0\le y\le h \quad t\le 0\\ &u=u_0t^n && \text{at} \quad y=h \quad t>0\\ &u=u_0t^n && \text{at} \quad y=0 \quad t>0 \end{aligned} \tag{1.7}$$

III. NUMERICAL COMPUTATION

Non-Dimensionalization of the Equations

The non dimensionalization of the governing equation is performed by selecting characteristic dimensionless quantities. The dimensionless quantities used in non dimensionalization of the governing equation (1.5) and the boundary condition (1.7) are

$$y^*=\frac{y}{h}, \qquad u^*=\frac{uh}{\upsilon} \quad \text{and} \quad t^*=\frac{t\upsilon}{h^2} \tag{1.8}$$

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial y^*}\frac{\partial y^*}{\partial t^*}\frac{\partial t^*}{\partial t}=\frac{\upsilon}{h}\frac{\partial u^*}{\partial t^*}\frac{\upsilon}{h^2}=\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*} \tag{1.9}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}=\frac{\upsilon}{h}\frac{\partial u^*}{\partial y^*}\frac{1}{h}=\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*} \tag{2.0}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right)=\frac{\partial}{\partial y^*}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y}=\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{2.1}$$

Replacing on the governing equation (1.6)

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\beta+\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2\left(u-u_0t\right)}{\rho} \tag{2.2}$$

Non dimensionalizing the relative velocity in equation (2.2) by setting $u^*=\frac{u}{\upsilon}h\Rightarrow u=\frac{\upsilon u^*}{h}$ and $t^*=\frac{t\upsilon}{h^2}\Rightarrow t=\frac{t^*h^2}{\upsilon}$ Substituting in (2.2) to non-dimensionalize the relative velocity

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\beta+\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right) \tag{2.3}$$

and multiplying the equation by $\frac{h^3}{\upsilon^2}$ gives

$$\frac{h^3}{\upsilon^2}.\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+\frac{h^3}{\upsilon^2}.V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{h^3}{\upsilon^2}.\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e{}^2H_0{}^2}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right) \tag{2.4}$$

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$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^* h^2}{\upsilon}\right) \tag{2.5}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}}{\rho}.\frac{1}{h}\left(\upsilon u^*-u_0\frac{t^* h^3}{\upsilon}\right) \tag{2.6}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}h^2}{\rho\upsilon^2}\left(\upsilon u^*-u_0\frac{t^* h^3}{\upsilon}\right) \tag{2.7}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}h^2}{\rho\upsilon^2}.\upsilon\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \tag{2.8}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}h^2}{\rho\upsilon}\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \tag{2.9}$$

The expression $\frac{\sigma\mu_e^{\ 2}H_0^{\ 2}h^2}{\rho\upsilon}=M^2$ is the Hartmann number squared, and $\frac{u_0 h}{\upsilon}$ is the Reynolds number Re and hence substituting in Equation 2.9, this gives

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \tag{3.0}$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-\frac{\mathrm{Re}\,h}{\upsilon}t^*\right) \tag{3.1}$$

Equation (3.1) is the governing equation in non-dimensional form.
Dimensionalizing the boundary conditions from (1.7) using the non-dimensional parameters from equations (1.9), (2.0) and (2.1) are obtained as

$$\begin{aligned} &u^*=0 \qquad 0\le y\le 1 \text{ and } t^*\le 0\\ &u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=1; \quad t^*>0\\ &u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=0; \quad t^*>0 \end{aligned} \tag{3.2}$$

Case II. Movement of the plates in the opposite direction (i.e. *n* = 1)

For case (ii) where the parallel porous plates of the channel are in motion in the opposite directions.
The governing equation is given by

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-\frac{\mathrm{Re}\,h}{\upsilon}t^*\right) \tag{3.3}$$

The boundary conditions are as follows

$$\begin{aligned} &u^*=0 \qquad 0\le y\le 1 \text{ and } t^*\le 0\\ &u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=1; \quad t^*>0 \end{aligned} \tag{3.4}$$

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$$u^* = -\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at} \quad y^* = 0 \; ; \qquad t^* > 0$$

The governing equations in non-dimensional form together with the boundary conditions for both cases will be presented in their finite difference forms consistent with the method of solution.

Governing Equation in Finite Difference Form

The finite difference analogues of the PDEs arising from the equation governing this flow are obtained by replacing the derivatives in the governing equations by their corresponding difference approximation .The following substitutions are done for the derivatives for the Crank Nicolson, we have the proposed averages as

$$u^* = \frac{u_{i,j+1} + u_{i,j}}{2} \tag{3.5}$$

$$\frac{\partial u^*}{\partial t^*} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \tag{3.6}$$

$$\frac{\partial u^*}{\partial y^*} = \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta y)} \tag{3.6}$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \tag{3.7}$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{1}{2}\left(\left\{\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2}\right\} + \left\{\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2}\right\}\right) \tag{3.8}$$

Replacing in the governing equation

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} + \frac{V_0 h}{\upsilon}\left(\frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta y)}\right) = \frac{h^3}{\upsilon^2}\beta +$$
$$\left[\frac{1}{2}\left(\left\{\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2}\right\} + \left\{\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2}\right\}\right)\right]$$
$$-M^2\left(\frac{u_{i,j+1} + u_{i,j}}{2} - \frac{\mathrm{Re}\, h}{\upsilon} t_j\right) \tag{3.9}$$

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} + \frac{V_0 h}{\upsilon}\left(\frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta y)}\right) = \frac{h^3}{\upsilon^2}\beta +$$
$$\frac{1}{2(\Delta y)^2}\left[\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) + \left(u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}\right)\right] \tag{4.0}$$
$$-M^2\left(\frac{u_{i,j+1} + u_{i,j}}{2} - \frac{\mathrm{Re}\, h}{\upsilon} t_j\right)$$

Multiplying through by Δt

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$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{\upsilon}\left(\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{4(\Delta y)}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left[\left(u_{i-1,j}-2u_{i,j}+u_{i+1,j}\right)+\left(u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right)\right] \qquad (4.1)$$
$$-M^2\Delta t\left(\frac{u_{i,j+1}+u_{i,j}}{2}-\frac{\mathrm{Re}\,h}{\upsilon}t_j\right)$$

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{4(\Delta y)\upsilon}\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left[u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}\right]-\frac{M^2\Delta t}{2}\left(u_{i,j+1}+u_{i,j}\right)+$$
$$M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}t_j \qquad (4.2)$$

Rearranging (4.2) gives

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-\frac{M^2\Delta t}{2}\left(u_{i,j+1}+u_{i,j}\right)+$$
$$M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}t_j \qquad (4.3)$$

Here $\gamma=-\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}$, $\zeta=\frac{h^3\Delta t}{\upsilon^2}\beta$, $\varsigma=\frac{1\Delta t}{2(\Delta y)^2}$, $\eta=\frac{M^2\Delta t}{2}$, $\vartheta=M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}$ and the suction/ injection parameter $S=\frac{V_0 h}{\upsilon}$.

Substituting the values of γ, ζ, ς, η, ϑ and S in (4.3) gives

$$\left(u_{i,j+1}-u_{i,j}\right)-\gamma\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\zeta+$$
$$\varsigma\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-\eta\left(u_{i,j+1}+u_{i,j}\right)+$$
$$\vartheta t_j \qquad (4.4)$$

Rearranging (4.4) gives

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$$u_{i,j+1} - u_{i,j} - \gamma u_{i+1,j+1} + \gamma u_{i-1,j+1} - \gamma u_{i+1,j} + \gamma u_{i-1,j} = \zeta +$$
$$\varsigma u_{i+1,j+1} - 2\varsigma u_{i,j+1} + \varsigma u_{i-1,j+1} + \varsigma u_{i+1,j} - 2\varsigma u_{i,j} + \varsigma u_{i-1,j}$$
$$-\eta u_{i,j+1} - \eta u_{i,j} + \vartheta t_j \tag{4.5}$$

Rearranging equation (4.5) gives

$$u_{i,j+1} - \gamma u_{i+1,j+1} + \gamma u_{i-1,j+1} - \varsigma u_{i+1,j+1} + \varsigma u_{i-1,j+1} + 2\varsigma u_{i,j+1} + \varsigma u_{i,j+1} = \zeta +$$
$$u_{i,j} - \gamma u_{i-1,j} + \gamma u_{i+1,j} + \varsigma u_{i+1,j} - 2\varsigma u_{i,j} + \varsigma u_{i-1,j} - \eta u_{i,j} + \vartheta t_j \tag{4.6}$$

Collecting the like terms form equation (4.6) gives

$$(1+2\varsigma+\eta)u_{i,j+1} - (\gamma+\varsigma)u_{i+1,j+1} + (\gamma+\varsigma)u_{i-1,j+1}$$
$$= \zeta + (1-2\varsigma+\eta)u_{i,j} + \varsigma u_{i+1,j} + (\varsigma-\gamma)u_{i-1,j} + \vartheta t_j \tag{4.7}$$

Rearranging equation (4.7)

$$-(\gamma+\varsigma)u_{i+1,j+1} + (1+2\varsigma+\eta)u_{i,j+1} + (\gamma+\varsigma)u_{i-1,j+1} =$$
$$\varsigma u_{i+1,j} + (1-2\varsigma+\eta)u_{i,j} + (\varsigma-\gamma)u_{i-1,j} + \vartheta t_j + \zeta \tag{4.8}$$

The finite difference equations obtained at any space node, say, i at the time level t_{j+1} has only three unknown coefficients involving space nodes at $i-1$, i and $i+i$ at t_{j+1}. In matrix notation, these equations can be expressed as $AU = B$ where U is the unknown vector of order $(N-1)$ at any time level t_{j+1}. B is the known vector of order $(N-1)$ which has the value of U at the n^{th} time level and A is the coefficient square matrix of order $(N-1)\times(N-1)$ which is a tridiagonal structure.

The coefficients of the interior nodes will be represented as:

$$\begin{aligned} a_j &= -(\gamma+\varsigma) & d_j &= (\varsigma-\gamma)u_{i-1,j} & g_j &= \vartheta t_j \\ b_j &= (1+2\varsigma+\eta) & e_j &= (1-2\varsigma-\eta)u_{i,j} & h &= \zeta \\ c_j &= (\gamma+\varsigma) & f_j &= \varsigma u_{i+1,j} & & \end{aligned} \tag{4.9}$$

For $j = 2,3,4....(N-1)$, then the equation (4.8) becomes

$$a_j u_{i+1,j+1} + b_j u_{i,j+1} + c_j u_{i-1,j+1} = d_j + e_j + f_j + g_j + h \tag{5.0}$$

The system of equations resulting from equation (5.0) are represented in a tridiagonal matrix form as

$$\begin{bmatrix} a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1} \end{bmatrix} \begin{bmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{3,j+1} \end{bmatrix} = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix} + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} f_2 \\ f_2 \\ \vdots \\ f_{N-1} \end{bmatrix} + \begin{bmatrix} g_2 \\ g_3 \\ \vdots \\ g_{N-1} \end{bmatrix} + \begin{bmatrix} h \\ h \\ \vdots \\ h \end{bmatrix} \tag{5.1}$$

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IV. RESULTS AND DISCUSSION

The physical situation of the problem and the effects of various flow parameters on the flow regime are depicted graphically and discussed. The simulations are curried out using ISO FLUIDS 3448 which are industrial oils whose kinematic viscosities range between 2 and 10.A constant pressure gradients between 1 and 5 and Reynolds numbers 1.The results are as follows;

Case 1: Plates moving in the same direction

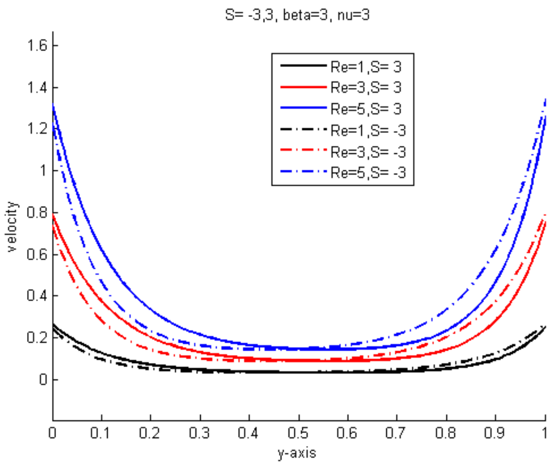
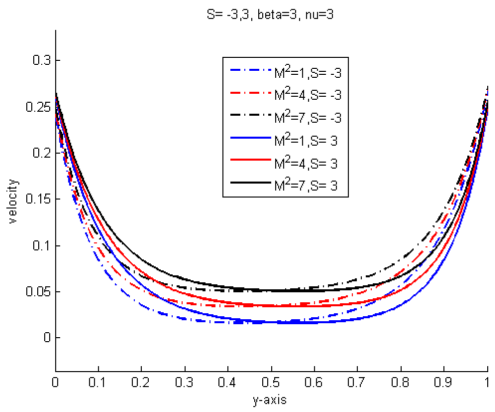
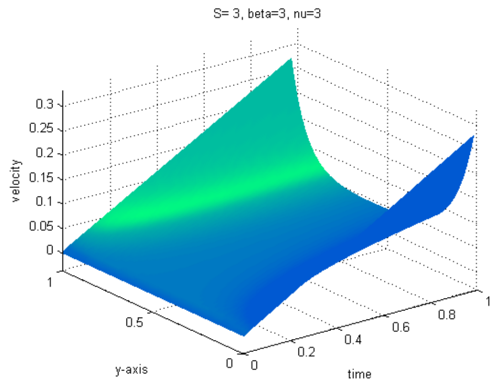


Fig.2: Varying the Reynolds number

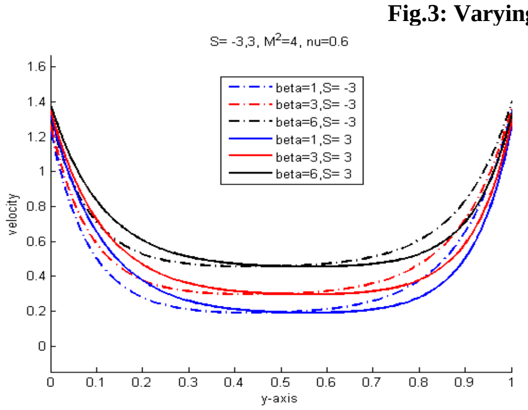


(a)

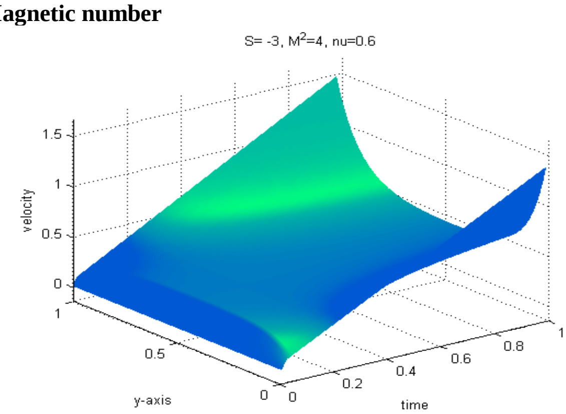


(b)

Fig.3: Varying the Magnetic number



(a)



(b)

Fig.4: Varying the Pressure gradient

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Case II: Plates moving in the opposite directions

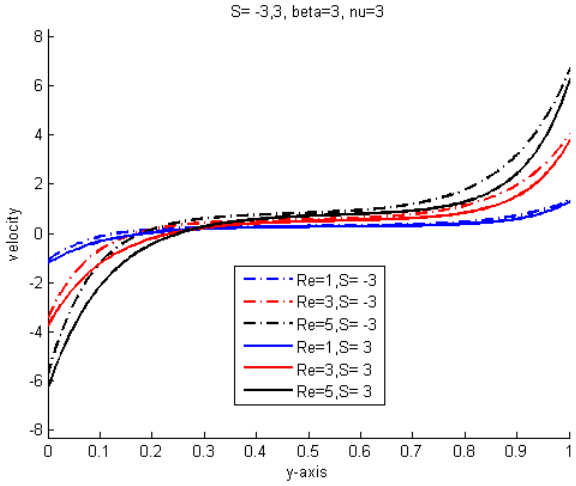
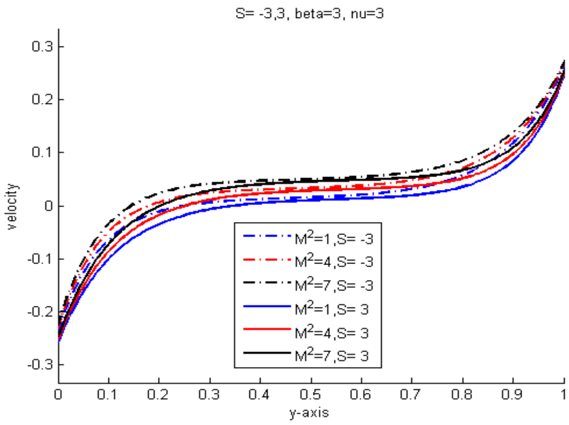
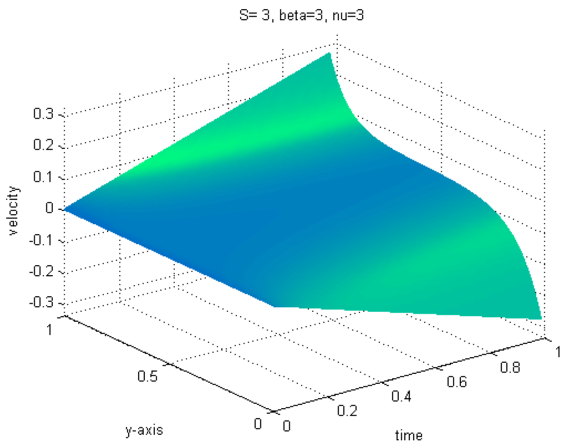


Fig.5: Varying the Reynolds number

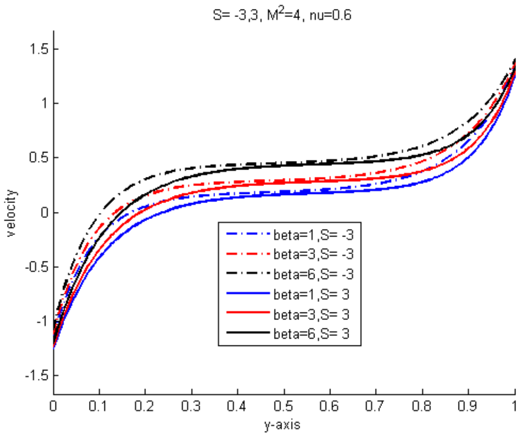


(a)

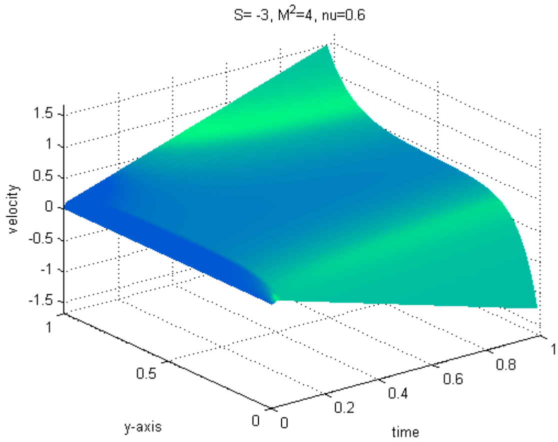


(b)

Fig.6: Varying the Magnetic number



(a)



(b)

Fig.7: Varying the Pressure gradient

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From figure 2. and figure 5. The velocity of the fluid increases with the increase in the Reynolds number for both suction on the upper plate and injection on the lower plate and injection on the upper plate and suction on the lower plate. The increase in the velocities are more pronounced at the plates due to the no-slip condition.

From figure 3. and figure 6. The increase in the magnetic number leads to an increase in the velocity of the fluid in the case where fluid suction is done on the upper plate and injection on the lower plate and the case where fluid injection is done on the upper plate and suction on the lower plate since the increase in the magnetic number reduces the drag force in the fluid hence increased velocities. The magnetic number gives a measure of the relative importance of the drag force resulting from the magnetic induction and the viscous forces in the fluid.

From figure 4. and figure 7. The velocity profiles increase with increase in the pressure gradient in both cases where fluid suction is done on the upper plate and injection on the lower plate and where fluid injection is done on the upper plate and suction on the lower plate since Pressure gradient is applied in the direction of the flow hence an increase in pressure gradient results in an increase in the force in the fluid in the direction of the flow which results in increased velocity profiles for the fluid flow.

For all the cases, it is observable that the velocities near the boundary for the plate with injection are greater than the velocities of the fluid near the plate with suction since injection of the fluid through the plate destabilizes the boundary, increasing the pressure and leading to a decrease in the viscous forces hence increase in the motion of the fluid.

V. CONCLUSION

The findings of this study leads to a conclusion that the magnetic field, pressure gradient, time and injection have an accelerating influence on the fluid flow with a constant pressure gradient in the direction of the flow on both cases of suction and injection. The injection and suction of fluid from either of the plates has a significant effect on the velocity profiles with injection leading to increased velocities and suction leading to decreased velocities of the fluid.

NOMENCLATURE

B Magnetic field strength vector, [wbm^{-2}]

H_o Magnetic flux identity along the y- axis [wbm^{-2}]

g Acceleration due to gravity vector, [ms^{-2}]

H Magnetic field intensity vector in Amperes per meter, [Am^{-1}]

J Current density, [AM^{-2}]

e Unit electric charge, [C]

E Electric field, [v]

S Suction/ Injection

M Magnetic parameter

Xi Permeability parameter

Re Reynolds number

P Pressure force, [nm^{-2}]

P^* Dimensionless pressure force.

q Velocity vector, [ms^{-1}]

i, j, k Unit vector is the x, y, z directions respectively

u, v, w Component of velocity vector q, [ms^{-1}]

u^*, v^*, w^* Dimensionless velocity components

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x^*, y^*, z^* Dimensionless Cartesian co-ordinates

x, y, z Dimensional Cartesian co-ordinates

F_i Body forces tensor, [N]

U_i Velocity tensor, [ms^{-1}]

x_j Space tensor, [m]

V_o Suction velocity, [ms^{-1}]

Greek symbol ***Meaning***

ρ Fluid density, [kgm^{-3}]

μ Coefficient of viscosity, [$kgm^{-1}s$]

σ Electrical conductivity, [Ω^{-1} m^{-1}]

μ_e Magnetic permeability, [Hm^{-1}]

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