# MACHAKOS UNIVERSITY

**University Examinations 2021/2022**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF PHYSICAL SCIENCES**

**THIRD YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR**

**BACHELOR OF EDUCATION (SCIENCE)**

**SPH 311: IONOSPHERIC**

**DATE: 17-3-2022** **TIME:2-00 – 4:00 PM**

**Useful constants**

Charge of an electron and mass of an electron e = 1.6 x $10^{-19}C$ m = 9.1 x $10^{-31}\,kg$ respectively, earth's magnetic field B = 5 x $10^{-5}Wb/m^2$ or 0.5 G or 0.5X10Wb/$cm^{-2}$. $\mu_o$ is the permeability of free space = $4\pi\ x\ 10^{-7}T.m/A$

**QUESTION ONE**

a) With an aid of a diagram define an electromagnetic wave (3 marks)

*An em wave comprise of an electric and a magnetic wave, at right angle to each other as shown in figure 1*

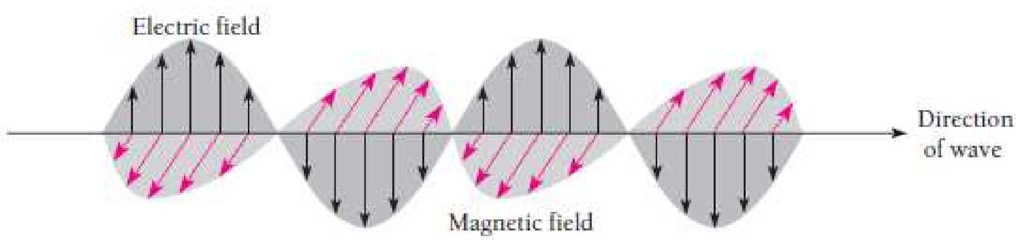



Figure 2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of propagation of the wave.

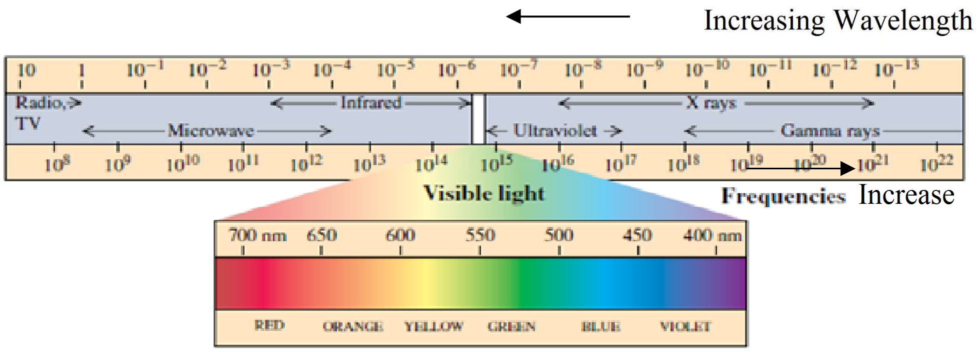
*figure 1*

b) Draw the electromagnetic spectrum showing (2 marks)

i. which way frequency and wavelength increase. (2 marks)

ii. position of radio waves (1 mark)

*solution*




c) Explain how density of air in the atmosphere changes as altitude decrease (3 marks)

*As altitude decrease density of air increase*

d) Explain what is meant by a "radio widow" (2 marks)

*The main frequency ranges allowed to pass through the atmosphere are referred to as the radio window.*

e) State two factors that determines whether an electromagnetic wave penetrates the atmosphere or not. (2 marks)

*Wave Frequency and ion density of the medium*

f) Distinguish between ground and sky waves (4 marks)

*The ground wave is the wave strongly influenced by the earth's surface. We can divide the ground wave into three components-earth reflected, direct and surface wave*

*"Sky waves" is a term used to describe propagation modes that use the earth's ionosphere.*

g) Define the following terms

i. Skip distance (2 marks

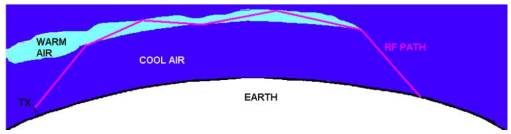
*the least distance between point of transmission and the point of reception*

ii. MUF (maximum usable frequency) (3 marks)

*the highest frequency that will be reflected back to earth by the ionized layers. Above this frequency there is no reflection and thus no skip.*

iii. Duct propagation (3 marks)

*Certain weather conditions produce a layer of air in the Troposphere that will be at a higher temperature than the layers of air above and below it. Such a layer will provide a "duct" creating a path through the warmer layer of air which has less signal loss than cooler layers above and below*




iv. Magneto-Ionic Theory (2 marks)

*Magneto-ionic theory is all about Motion of Ions in Electric and Magnetic Fields. The theory of the propagation of electromagnetic waves by an ionized medium in an external magnetic field*

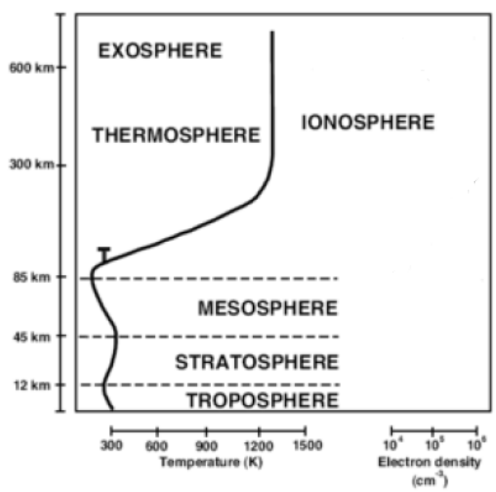
h) In 1899,.in a landmark experiment on December 12, 1901, Marconi, who is often called the "Father of Wireless," demonstrated transatlantic communication by receiving a signal in St. John's Newfound land that had been sent from Cornwall, England. Explain how this was possible (3 marks)

*This was made possible due to reflection of radio waves by the ionosphere*

**QUESTION TWO**

a) Using a diagram, show how temperature of the atmosphere varies with height (3 marks)

*solution*

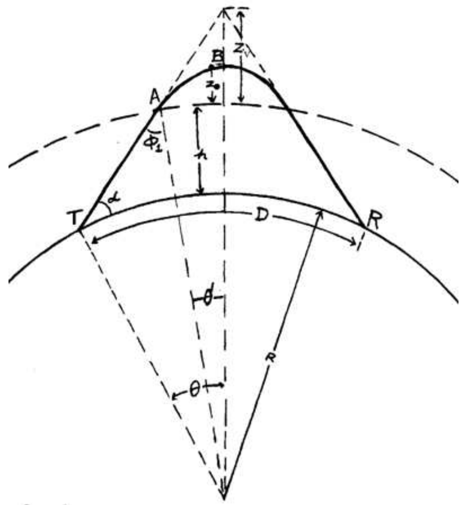



b) Define an ionosode and give the equation used to determine virtual height (3 marks)

*An ionosonde is an equipment designed to measure directly the time t taken for a pulse of radio waves to travel up to the ionosphere and back as a function of frequency*

c) Distinguish virtual height from actual height using a diagram (4 marks)

*solution*




*where $Z_o$ actual height and $Z_v$ is virtual height*

d) The relationship between pressure P and density $\rho$ at any height h is given by the "barometric equation,"

i. Derive the barometric equation (8 marks)

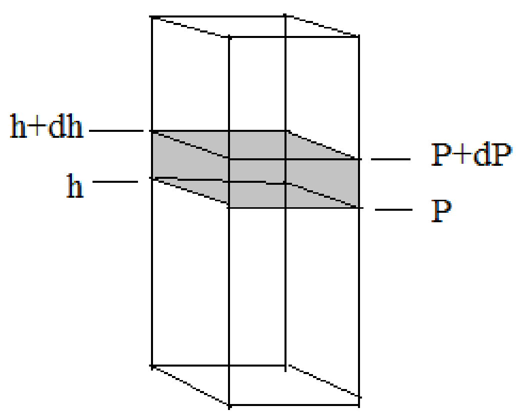
ii. Consequently, define Scale height (2 marks)

*solution*

*i.* $F = mg = \rho gV = \rho ghA$ *where $\rho$ is the gas density. Then the gas pressure becomes*

$P = \frac{F}{A} = \frac{\rho ghA}{A}$ *implying* $P = \rho gh$ *1*

*Now imagine such a column in the atmosphere and separated by a thin layer with height dh*




*iii.*

*Such a layer causes the pressure to change by the value of*

$dP = -\rho gdh$ *the minus sign mean pressure decrease as altitude increase*

*2*

*Considering atm air asideal gas, we can use the ideal gas law to express the density $\rho$ in terms of pressure.*

$PV = \frac{mRT}{M}$

$P = \frac{mRT}{VM}$

$P = \frac{\rho RT}{M}$ *where T is absolute temperature, R is the universal gas constant, M is the molar mass, which is for air. Therefore the density is given by the formula*

$\rho = \frac{MP}{RT}$ *3*

*Substituting in equation 2*

$dP = -\frac{MP}{RT} gdh$ *4*

$\frac{dP}{P} = -\frac{M}{RT} gdh$ *5*

$\frac{dP}{P} = -\frac{dh}{H}$ *where* ***H =*** $\boldsymbol{\frac{RT}{Mg}}$ ***is called scale height***

*Equation 5 is a differential equation describing the gas pressure P as a function of altitude h. Integrating gives the equation*

$\int \frac{dP}{P} = -\int \frac{M}{RT} gdh$ *integration by separation of variables*

*In P* = $-\frac{M}{RT} gh + In\ C$ *std intg* $\int \frac{du}{u} = In\ u$

*Getting rid of the logarithms we get*

$P = C\ exp(-\frac{M}{RT} gh)$ *since exp 0 =1*

*The constant of integration C can be determined from the initial condition* $P(h = 0) = P_0$

*Barometric equation becomes*

$$P = P_0\ exp(-\frac{M}{RT} gh)$$

**QUESTION THREE**

a) Explain why at night, even after disappearance of the sun, the ionosphere is continued to be charged, although not as strongly as during the day. (4 marks)

*Because of presence of cosmic rays from the outer space*

b) Explain how man made electromagnetic waves (radio waves) can be generated, transmitted and be received over a large distance (6 marks)

*In a microphone, a coil in a magnet is made to vibrate when a person speaks on a diaphragm, this generates an em signal, the varying signal makes electrons in a transmission antennae oscillate, as they do so, an em wave is radiated, which travelsto a receiving antennae, where again the vibrate electrons, setting up a signal that can be converted to sound by a loud speaker.*

c) Write down the continuity equation in terms of electron effective rates of production and disappearance. (2 marks)

$$\frac{dN}{dt} = q_{eff} - \alpha_{eff} N^2,$$

d) Explain why the

i. The D layer reduces after sunset but remains at night (2 marks)

*The layer reduces greatly after sunset because the primary source of ionization is not there and here rate of recombination is high, but remains due to galactic cosmic rays.*

ii. E layer starts to disappear after sun set (2 marks)

*At night the E layer begins to disappear because the primary source of ionization is no longer present.*

e) Figure 1 shows the regions of the atmosphere. Redraw the figure and position the ionosphere, clearly showing the layers D, E , $F_1$ and $F_2$ (2 marks)

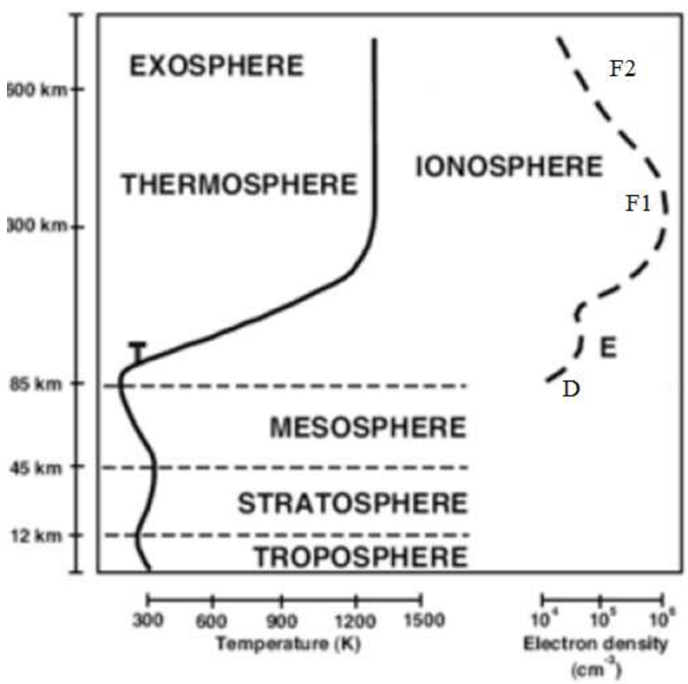
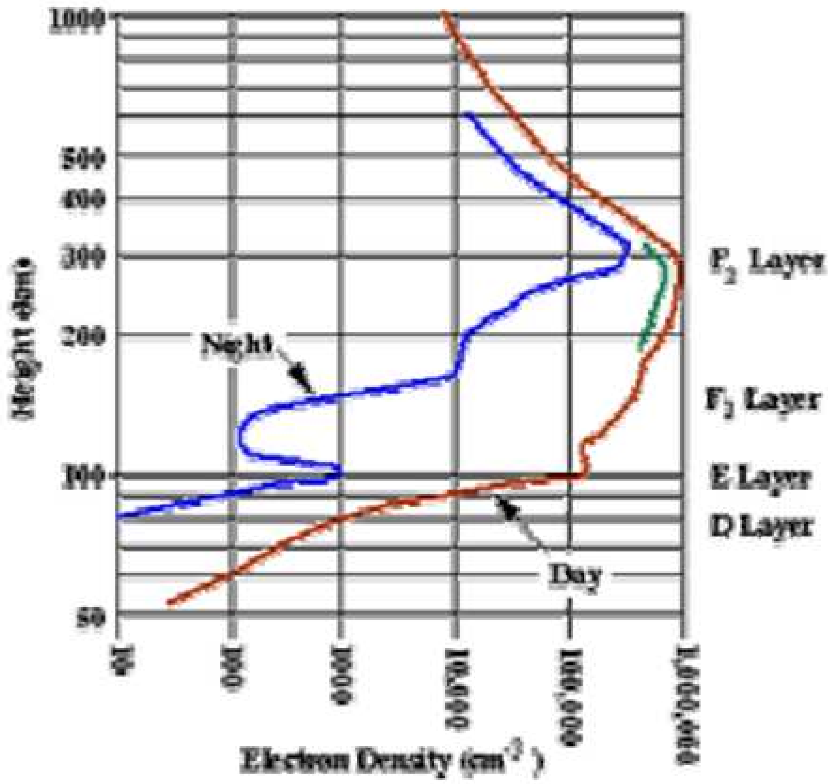



**Figure 1**

f) Draw a diagram showing electron profile density in the ionosphere as a function of height (altitude) for both day and night showing the D, E , $F_1$ and $F_2$ (2 marks)




**QUESTION FOUR**

a) An electron e moving with a velocity *v* in the earth's magnetic field B experience deflecting force F. What is the magnitude of this force on each electron in terms B, e and *v*. (2 marks)

*F = e(v x B)*

b) The numerical value of gyrofrequency $f_g$ is of great importance in radio wave propagation . Derive a mathematical expression for the gyrofrequency

$f_g = \left(\frac{1}{2\pi}\right)\left(\frac{\mu_o eH}{m}\right)$ (6 marks)

*Assume that an electron revolves in a circular path of radius r due to the perpendicular magnetic field of the earth. Hence necessary centripetal force is provided by Lorentz force*

$F = \frac{mv^2}{r}$

*We get* $\frac{\boldsymbol{v}}{\boldsymbol{r}} = \frac{\boldsymbol{eB}}{\boldsymbol{m}}$ *Since* $\frac{v}{r} = \ \omega = 2nf$

$f_g = \left(\frac{1}{2\pi}\right)\left(\frac{eB}{m}\right) = \left(\frac{1}{2\pi}\right)\left(\frac{\mu_o eH}{m}\right)$ *Where* $B = \mu_o H$

*All*

*centrifugal force = centripetal force*

$mr_h\omega_H^2 = er_H\omega_H B$ 11

*hence.*

$\omega_H = \frac{e}{m} B$ 12

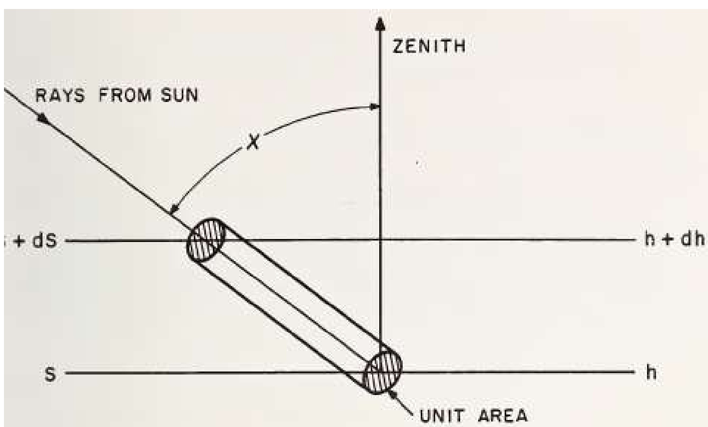
$f_H = \frac{1}{2\pi}\frac{e}{m} B$ 13

c) State 4 assumptions made in the formation of the chapman's layer (4 marks)

1. *An atmosphere with only one type of gas.*
2. *Plane stratification.*
3. *A parallel beam of monochromatic ionizing radiation from the sun.*
4. *An isothermal atmosphere.*

d) Derive the Chapman's formula (8 marks)

*Let the ionizing radiation of intensity be incident, at a zenith angle* χ,




on the top of the atmosphere. As the radiation is absorbed, as it penetrates into the atmosphere, its intensity diminishes. Let S be the energy flux at a height h and S +dS the flux at a height h + dh as sketched in figure 1.2. Let $\sigma$ be the absorption cross section of the atoms of the gas and N their number density. The energy absorbed dS in a cylinder of unit cross section and axis parallel to the direction of the incident beam is given by

$$dS = S\sigma N\, dh \sec \chi.$$

Upon integration we obtain

$$\int \frac{dS}{S} = \sec\chi \int N\sigma\, dh = -\tau \sec\chi, \tag{1.18}$$

where

$$\tau = -\int N\sigma\, dh \tag{1.19}$$

is the optical depth of the atmosphere down to the height $h$. The minus sign arises because $\tau$ increases in the opposite direction to $h$.

At great heights, $S \rightarrow S_\infty$ as $\tau \rightarrow 0$ and (1.18) gives

$$S = S_\infty \exp(-\tau \sec\chi). \tag{1.20}$$

The energy absorbed per unit volume is given by

$$\frac{dS}{dh \sec\chi} = N\sigma S = N\sigma S_\infty \exp(-\tau \sec\chi). \tag{1.21}$$

Let $\eta$ be the number of ion pairs produced per unit quantity of energy absorbed, i.e., the ionization efficiency. The number of ion pairs produced per unit volume per second is

$$q(\chi, h) = N\sigma S_\infty \eta \exp(-\tau \sec\chi). \tag{1.22}$$

Now $\tau$ is a function of $\chi$ and $h$. Using (1.1), (1.2), and (1.3), we obtain

$$\tau = \sigma \int N\, dh = \frac{\sigma}{mg} \int dp = \frac{\sigma p}{mg} = \frac{\sigma N k T}{mg} = \sigma N H. \tag{1.23}$$

Substitution of (1.23) into (1.22) yields

$$q(\chi, h) = \frac{\tau S_\infty}{H\epsilon} \eta \exp(1 - \tau \sec\chi), \tag{1.24}$$

$$q(\chi, z) = \frac{S_\infty \eta}{\epsilon H} \exp\{1 - z - \sec\chi \exp(-z)\} \qquad \text{(a)}$$

**QUESTION FIVE**

a) Show that equation of motion of an ion in an electric field is is

i. $\ddot{x}_2 + \frac{Ne^2}{\varepsilon_o m} x_2 = 0$ (8 marks)

ii. Hence define angular plasma frequency $\omega_N$ (2 marks)

*i.* *Solution*

The equation of motion of an ion of charge e and mass m in an electric field E V/m, is

$m\ddot{x} = eE$ 1 ( *recall* $F = qE$)

from which, by integration, we obtain the velocity

$\dot{x} = \frac{e}{m} E\, t$ 2

If the ion concentration is N (per cubic meter), the current density created is

$J = Ne\dot{x}$ 3

Substitute 2 into 3

$J = \frac{Ne^2}{m} Et$ 4

*Consider now a slab of plasma (a medium containing free charges) in which the light electrons and the heavy positive ions are displaced, as shown in figure 2.4, by a distance X2.*

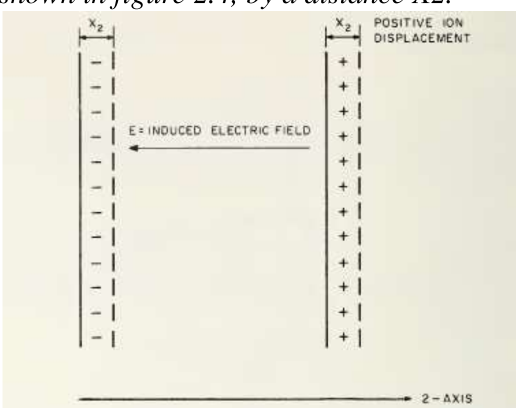



FIGURE 2.4. *Force on an electron due to plasma displacement.*

*The surface charge density is* $Nex_2$ *so that the field is given by*

$$E = -\frac{Ne}{\varepsilon_o} x_2$$

$m\ddot{x}_2 = eE = -\frac{Ne}{\varepsilon_o} x_2$ 6

Or

$\ddot{x}_2 + \frac{Ne^2}{\varepsilon_o m} x_2 = 0$ 7

*This is the equation of a simple harmonic motion with angular frequency* $\omega_N$

$\omega_N = \frac{Ne^2}{\varepsilon_o m}$ *8*

*where* $\omega_N$ *is called the angular plasma frequency.*

b) The motion of an ion in a the earth's magnetic field is a helix where centrifugal force is equal to centripetal force, or $mr_H\omega_H^2 = er_H\omega_H B$. Show that

i. $f_H = \frac{1}{2\pi}\,\omega_H$ where $f_H$ is called the gyro (or gyromagnetic) frequency and $\omega_H$ the angular velocity (3 marks)

ii. $f_H = 2.84\ x\ 10^{10}\ B$ (2 marks)

iii. $f_H = 3.57\ x\ 10^4\ H$ (3 marks)

iv. $f_H = 1.42\text{X}10^6\ \text{c}/m^{-1}$ (2 marks)

centrifugal force = centripetal force

$mr_H\omega_H^2 = er_H\omega_H B$

Hence making $\omega_H$ subject of the formula

$\omega_H = \frac{e}{m}\ B$

$f_H = \frac{1}{2\pi}\frac{e}{m}\ B$

Here $f_H$ is called the gyro (or gyromagnetic) frequency and $\omega_H$ the angular

Notice that $f_H$ is independent of the velocity of the ion. For electrons we have

$$f_H = 2.84\ x\ 10^{10}\ B = 3.57\ x\ 10^4$$

Now in the ionosphere, B is on the order of 0.5 G or 0.5X10Wb/$cm^{-2}$; therefore, $f_H$=1.42X$10^6$ c/$m^{-1}$ or 1.42 Mc/s.

.