MACHAKOS UNIVERSITY

University Examinations 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR

BACHELOR OF EDUCATION SCIENCE (SPECIAL NEEDS)

BACHELOR OF SCIENCE IN APPLIED PHYSICS AND TECHNOLOGY

BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS 1

DATE: 14/03/2022 **TIME: 8:30-10:30 AM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE

a) What is the smallest possible uncertainty in the position of an electron moving with velocity 10^6 m/s? (3 marks)

b) Show that $[x, E_x] = -i\hbar v$ (5 marks)

c) Show that for a one-dimensional square - integrable wave-packet,

$$\int_{-\infty}^{\infty} j(x)dx = \frac{\langle p \rangle}{m}$$

Where $j(x)$, is the probability current? (7 marks)

d) State the mathematical operator for position, momentum and energy for a normalised wave function. (3 marks)

e) Write down expression for time independent Schrodinger equation in one dimension. (2 marks)

f) A particle of mass *m* is trapped in a one dimensional box with a potential described by

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{Otherwise} \end{cases}$$

Solve the Schrödinger equation for this potential. Taking $\Psi(x,t) = \Psi(x)e^{-iEt/\hbar}$ (8 marks)

g) Explain two failures of classical mechanics from the photoelectric experiment that led to quantum mechanics approach (4 marks)

QUESTION TWO

Prove that $\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}$ Where $\langle x\rangle$ and $\langle p\rangle$ are the mean values of the coordinate and momentum of the particle, respectively. (20 marks)

QUESTION THREE

a) State the mathematical equation for expectation values momentum, kinetic energy and hamilitonian as used in quantum mechanics (3 marks)

b) Show that the momentum operator $-i\hbar\frac{\partial}{\partial x}$ is Hermitian operator. Obtain eigenfunction of momentum operator (17 marks)

QUESTION FOUR

a) What are the two conditions for normalization of a wave function (2 marks)

b) Consider a particle described by a wave function *ρ(r, t)*. Calculate the time-derivative $\frac{\partial\rho(r,t)}{\partial t}$ where ρ*(r, t)* is the probability density and show that the continuity equation $\frac{\partial\rho(r,t)}{\partial t} + \nabla.\vec{J}(r,t) = 0$ is valid, where ***J****(r,t)* is the probability current

(18 marks)

QUESTION FIVE

a) Derive Time – dependent Schrödinger equation and hence write it in three dimension given that the wave equation is $\Psi = Ae^{i(kx-\omega t)}$ (18 marks)

b) The Schrödinger equation is a first order equation with respect to time discuss. (2 marks)