MACHAKOS UNIVERSITY

University Examinations 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF EDUCATION SCIENCE.

SPH 301: QUANTUM MECHANICS I

DATE: **TIME:**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

SECTION A

QUESTION ONE (30 MARKS)

a) The interpretation of $|\Psi(\vec{r},t)|^2$.as a position probability density requires that the probability be conserved. Explain the meaning of this statement. (2 marks)

b) What do you understand by the degree of degeneracy of an eigenvalue? Write down the time-independent Schrödinger equation as an eigenvalue equation. (2 marks)

c) Consider a square integrable wave function: $\Psi(\vec{r},t)$and a Hamiltonian operator H. What is the condition for the Hamiltonian to be Hermitian? (2 marks)

d) Show that the energy of a particle in a wave packet is a function of momentum and is given by $E(p_x) = \frac{p_x{}^2}{2m}$ where the parameters have their usual meaning. (3 marks)

e) Consider an operator $\hat{R} = \frac{-d^2}{dx^2}$ and the eigen value equation $\hat{R}u(x) = \lambda u(x)$. Write out the possible eigen functions and discuss the conditions under which they are well behaved.

(3 marks)

f) State Pauli's exclusion principle to prove that an atomic shell with quantum number n can accommodate only $2n^2$ electrons (4 marks)

(g) Define and explain the four electronic quantum numbers. (5 marks)

h) Write down the Gaussian wave packet wave function and explain the parameters in the function. What is the normalization condition for the wave function? (4 marks)

i) Discuss the two slit experiment that serves to demonstrate the wave particle duality of electromagnetic radiation. (5 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) What is a wave function? Discuss its role in the two slit experiment. (4 marks)

b) Discuss the meaning of Heisenberg uncertainty relationship, $\Delta x \Delta p_x \geq \hbar$. Explain its application in the two-slit experiment. (5 marks)

c) Prove that the Eigen values of Hermitian operators are real. (5 marks)

d) Separate the time-dependent Schrödinger equation $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$ into two equations, one equation depending on x alone and the other on t alone. (6 marks)

QUESTION THREE (20 MARKS)

a) Show that $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ is Hermitian (4 marks)

b) If ψ_1 and ψ_2 are eigenfunctions of an Hermitian operator, show that $\int \psi_1 {\psi_2}^* d\tau = 0$

(6 marks)

c) Show that the Schrödinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$ can be obtained from Helmholtz's equation $\frac{d^2\psi(x,t)}{dx^2} + k^2\psi(x) = 0$ by substituting $K = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{p}$

(5 marks)

d) If $u = Axe^{-x^2/2}$ is an eigenfunction of the operator $\hat{H} = \frac{-d^2}{dx^2} + x^2$, using the equation $\hat{H}u = \lambda u$ determine the value of the Eigen value λ (5 marks)

QUESTION FOUR (20 marks)

a) What is the essence of Quantum Mechanics? Explain using two examples which classical mechanics fails to account. (4 marks)

b) If the wave function $\psi(x,t)$ is linear show that the time-dependent Schrödinger equation is also linear in. $\psi(x,t)$ (6 marks)

c) Derive the infinite square-well energy quantization law $E = \frac{\pi^2 n^2 \hbar^2}{2ma^2}$ by using equation $p = \frac{h}{\lambda}$ and $a = n\frac{\lambda}{2}$ where $p = \frac{h}{\lambda}$ is de Broglie's relation and "a" is the width of the well. (6 marks)

d) Given the wave function $\psi(x) = Axe^{-x^2/2}$ calculate the expectation value of x. (4 marks)

QUESTION FIVE (20 MARKS)

a) A particle bounces back and forth between the walls of one-dimensional box at $\pm\frac{a}{2}$. The wave function for the lowest energy state of the particle is $\psi(x,t) = Acos\left(\frac{x\pi}{a}\right)e^{\left(\frac{-iEt}{\hbar}\right)}$ in the region $-\frac{a}{2} < x < \frac{a}{2}$ and it is zero outside this region. Show that it is a solution to the Schrödinger equation in the region and determine the value of energy E for the lowest energy state (10 marks)

b) Determine the energy levels and the corresponding normalized Eigen functions of a particle in one-dimensional potential well of the form:
$V(x) = \infty$ for $x < 0$ or for $x > a$; and $V(x) = 0$ for $0 < x < a$ (10 marks)