MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (HOSPITALITY AND TOURISM MANAGEMENT)

SMA 361: THEORY OF ESTIMATION

DATE: **TIME:**

INSTRUCTION:

ANSWER QUESTION ONE AND OTHER TWO QUESTIOS

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Explain the meaning of the following terms as applied in theory of estimation

(i) An estimator

(ii) Consistency (4 marks)

(b) A random sample of five values is taken from a normal population with mean μ and variance σ^2both unknown. The values are: 1.07, 1.02, 1.04, 1.06, and 1.05. Estimate the population mean and population variance. (4 marks)

(c) $Let\ x_1, x_2, .. x_n$ be a random sample drawn from a population with mean μ

(unknown).Show that if $y = \sum_1^n a_i x_i$ is unbiased estimator for μ if $\sum_1^n a_i = 1$

(5 marks)

(d) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $f(x, \lambda) = \lambda x^{\lambda - 1}$, 0$< x < 1, \lambda > 0$

Show that $t_1 = \prod_{i-1}^{n} x_i$ is sufficient for λ (5 marks)

(e) Consider a normal distribution with mean (μ) and variance σ^2 .Let $\mu_1 = \bar{x}_n = \sum_1^n \frac{x_i}{n}$ and $\mu_2 = \bar{x}_m = \sum_1^m \frac{x_i}{m}$ where m>n, be estimators of μ. Determine which estimator is more efficient (6 marks)

(f) Let $x_1, x_2, ..x_n$ be a random sample of size n from a distribution with mean from a population with mean μ (unknown) and known variance σ^2.Determine the maximum likelihood estimator (MLE) of σ^2 (6 marks)

QUESTION TWO (20 MARKS)

(a) $Let\ x_1, x_2, ..x_n$ be a random sample from a population with mean μ and variance δ^2.

Consider the following estimators forμ.

$$t_1 = {}^1/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
- Determine the efficiency of t_3 relative to t_2 (5 marks)

(b) Let X be random variable that is binomially distributed with a pdf defined as

$$f(x;\lambda) = \binom{n}{x}\lambda^x(1-\lambda)^{n-x} \quad for\ x = 0,1 \ldots n, \lambda \in (0,1).$$

Determine the Crammer-Rao lower bound (10 marks)

QUESTION THREE (20 MARKS)

(a) Use the following data to fit the regression line $y = \propto + \beta x$

x	-2	-1	0	1	2
y	0	0	1	1	3

(8 marks)

(b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased. (12 marks)

QUESTION FOUR (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\lambda + 1)x^{\lambda}, & 0 < x < 1, \ \lambda > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of λ (8 marks)

(b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x,\theta) = \frac{e^{-\theta}\theta^{x}}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\theta}$ (12 marks)

QUESTION FIVE (20 MARKS)

(a) Given that 8.6,7.9,8.3,6.4,8.4,9.8,7.2,7.8,7.5 are observed values of a random variable which is distributed normally with mean 8 and unknown variance σ^2.Given that $\chi^2_{\frac{\alpha}{2},n} = 19.2 \ and \ \chi^2_{1-\frac{\alpha}{2},n}$=2.70.construct a 95% confidence interval for σ^2 (10 marks)

(b) Let $x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf if $f(x, \theta)$where θ is unknown. Let $T = t(x_1, x_2, ... x_n)$ be the necessary and sufficient

condition for T to be the Minimum variance unbiased estimator (MVUE) of θ expressed in the form

$$\frac{\partial LogL}{\partial \theta} = \frac{T - \theta}{\lambda}$$

Show that T is MVUE of θ and $Var(T) = \lambda$ (10 marks)