MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2022/2023

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR BACHELOR OF EDUCATION, BACHELOR OF SCIENCE AND BACHELOR OF ARTS

SMA 202: LINEAR ALGEBRA I

DATE: **TIME:**

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Considering mathematical context, define the following vector space terms;

i) Vector subspace (1 mark)

ii) Spanning sets (1 mark)

iii) Linear dependence of vectors (1 mark)

iv) Basis (1 mark)

b) Use the Cramer's rule to solve the systems of equations;

$$2x + 3y = 1$$
$$5x + 7y = 3$$
(4 marks)

c) A stable floor model of an aircraft is determined by the points $A(2,\ -1,\ 1),\quad B(3,\ 2,\ -1),\quad C(-1,\ 3,\ 2)$. Calculate its equation. (4 marks)

d) Show that the following vectors form a right angled triangle.

$$\vec{a} = 4i - j + k$$
$$\vec{b} = 3i - 2j - k$$
$$\vec{c} = i + j + 2k$$
(4 marks)

e) Determine the vector projection of the point $A(2,\ 7)$ $on \quad the \quad po$int $B(-3,\ 1)$ (4 marks)

f) Consider the matrix;

$$S=\begin{pmatrix}10 & 7 & 8 & 7\\ 7 & 5 & 6 & 5\\ 8 & 6 & 10 & 9\\ 7 & 5 & 9 & 10\end{pmatrix}$$

Reduce S to its Echelon form (5 marks)

g) Consider the matrix

$$A=\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 1 & 5 & 7\end{pmatrix}$$

Determine A^{-1} (5 marks)

QUESTION TWO (20MARKS)

a) Let ; $u = 2i - 3j + 4k, v = 3i + j - 2k$, Find $(u + v).(2u - 3v + 4w)$.
(5 marks)

b) Express $v = (2, -5,3) in\ \mathbb{R}^3$ as a linear combination of the vectors
$u_1 = (1, -3,2), u_2 = (2, -4, -1), u_3 = (1, -5,7)$. (5 marks)

c) Determine whether or not the vectors $u = (1,1,2), v = (2,3,1), w = (4,5,5)\ in\ \mathbb{R}^3$ are linearly dependent. (5marks).

d) Prove that $H = \begin{pmatrix} cos\emptyset & 0 & sin\emptyset \\ sin\theta sin\emptyset & cos\theta & -sin\theta cos\emptyset \\ -cos\theta sin\emptyset & sin\theta & cos\theta cos\emptyset \end{pmatrix}$ is an orthogonal matrix. (5 marks)

QUESTION THREE (20MARKS)

a) Determine the angle between (4 marks)
$\tilde{u} = i + j + k$ and $\tilde{v} = i + j - k$

b) Consider the parallelogram whose adjacent sides are given by
$2i - 4j + 5k$ *and* $i - 2j - 3k$. Calculate;
   i) Unit vector parallel to its longer diagonal (5 marks)
   ii) Area of the parallelogram (5 marks)

c) Using reduction to echelon form solve
$$x + y - z = 7$$
$$x - y + 2z = 3$$

$$2x + y + z = 9$$

(6 marks)

QUESTION FOUR (20MARKS)

a) Calculate the equation of a plane passing through point (2,1, 3) given that it is perpendicular to the vector $5i + 6j + 7k$ (4 marks)

b) Determine the parametric equation and vector equation of a line whose Cartesian equation is given by;

$$\frac{x-1}{5} = \frac{y+1}{2} = \frac{z}{-5}$$ (6 marks)

c) Determine the acute angle between lines; $\frac{x-1}{5} = \frac{y+2}{4} = \frac{z}{6}$ and $\frac{x+3}{2} = \frac{y-5}{7} = \frac{z+1}{-1}$ (10 marks)

QUESTION FIVE (20MARKS)

a) Determine λ such that $\tilde{a} = i + j + k, \tilde{b} = 2i - 4k \ and \ \tilde{c} = i + \lambda j + 3k$ are coplanar. (4 marks)

b) Determine the values of x that satisfy the equation: $\begin{vmatrix} x & 3+x & 2+x \\ 3 & -3 & -1 \\ 2 & -2 & -2 \end{vmatrix} = 0$ (6 marks)

c) If $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}, N = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, show that $(MN)^{-1} = N^{-1}M^{-1}$. (10 marks)