



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 103: ANALYTICAL GEOMETRY

DATE:

TIME:

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Calculate the equation of the straight line which passes through the point of intersection of the lines $4x - 3y + 19 = 0$, $12x - 5y + 3 = 0$ and makes an intercept of $+ 2$ on the $x -$ axis (5 marks)
- b) Find the equation of the locus of the point that moves such that its distance from point A (3,1) is twice the distance from the point B(5,1). (5 marks)
- c) Calculate the focus, equation of directrix and length of latus rectum of the parabola $5y^2 = 24x$ (5 marks)
- d) Convert the Cartesian coordinates of the point $(-4, 3)$ into Polar coordinates. (5 marks)
- e) Determine the radius and the coordinates of the centre of the circle given by the equation $x^2 + y^2 - 8x - 2y + 8 = 0$ (5 marks)
- f) Calculate the eccentricity of the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = -1$ at the point $(3, \frac{12}{9})$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Determine the parametric equation of the parabola $y^2 - 12y - 20x + 76 = 0$. (4 marks)
- b) Determine the vertex, the axis of symmetry, the focus, the equation of the directrix and the length of the latus of the parabola $x^2 - 10x - 6y = -40$ (6 marks)
- c) Determine the point on the directrix of the hyperbola $x = 7\sec\theta, y = 2\tan\theta$ which the tangent at the point $(7\sec60, 2\tan60)$ passes through. (10 marks)

QUESTION THREE (20 MARKS)

- a) Determine the polar equation of the parabola $x^2 = -12(y - 3)$ (5 marks)
- b) Determine the equation of the ellipse whose focus is $\frac{2}{3}$ and the directrix is $y = 9$ (5 marks)
- c) Prove that the equation of the chord joining two points whose eccentric angles are θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is given by $\frac{x}{a} \cos\left(\frac{\theta_1 + \theta_2}{2}\right) + \frac{y}{b} \sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 - \theta_2}{2}\right)$ (10 marks)

QUESTION FOUR (20 MARKS)

- a) Calculate the distance from the point $(5, 8)$ to the circle $x^2 + y^2 + 2x + 6y + 3 = 0$ and state whether the point is outside, inside or on the circle. (5 marks)
- b) Determine the angle between the lines $y = 2x + 5$ and $y = 12x - 1$ (5 marks)
- c) Determine the equation of the tangent to the circle $y^2 + x^2 - 5 = 0$ from the point $(8,3)$ on the circle. (10 marks)

QUESTION FIVE (20 MARKS)

- a) Determine the point on the parabola $y^2 - 12y - 14x + 162 = 0$ through which the tangents which meet at $Q(4,2)$ pass (4 marks)
- b) Determine the equation of the tangent to the parabola $x^2 - 22x + 24y + 169 = 0$ at the point whose focal distance is 11 units (6 marks)
- c) Consider the hyperbola whose equation is given as $y^2 - x^2 - 2y + 4x - 7 = 0$. Sketch the hyperbola labelling foci and vertices. (10 marks)