MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (METHEMATICS)

BACHELOR OF EDUCATION(SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: **TIME:**

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Obtain a partial differential equation of first order by eliminating the arbitrary functions of the equation $u = e^{-y}f(x) + e^{x}g(y)$ (5 marks)

b) Show that the Laplace's equation in three dimensions $u_{xx} + u_{yy} + u_{zz} = 0$ is satisfied by the function $u = r^{-1}$ where;
$u = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ (5 marks)

c) Obtain the solution to the initial value problem $u_{xx} = 4xy + e^{x}$ with initial conditions $u(0, y) = y, u_x(0, y) = 1$ (6 marks)

d) Solve the following boundary value problem
$u_{xy} = 4xy + e^{x}; u_y(0, y) = y, u(x, 0) = 2$ (6 marks)

e) An infinitely long string having one end at $x = 0$ is initially at rest on the $x - axis$. At $t = 0$, the end $x = 0$ begins to move along the $u - axis$ in a manner described by $u(0, t) = a\cos\sigma t$. Find the displacement $u(x, t)$ of the string at any point at any time (8 marks)

QUESTION TWO (20 MARKS)

a) Consider the following equation, $u_{xx} + 4u_{xy} + 4u_{yy} = 0$. Find the differential equation of the characteristic curve and obtain the general solution (4 marks)

b) Fourier transform of a function $f(x)$ if it exists is defined by

$$\bar{F}(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\xi x}dx$$

Hence or otherwise find the Fourier transform of the function

$f(x) = e^{-ax^2}; a > 0$ (6 marks)

c) Show that the cylindrical polar form of a Laplace's equation in two dimensions is given by; $\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}$ where $u(r,\theta)$, $r(x,y)$ and $\theta(x,y)$ (10 marks)

QUESTION THREE (20 MARKS)

a) Find the Riemann-Green function for the equation

$\frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - 2u = 0$ and hence obtain the solution that satisfies the conditions $u = 0, \frac{\partial u}{\partial x} = 2x$ on $y = x$ (10 marks)

b) Consider the following nonlinear second order partial differential equation, $r - t\cos^2 x + p\tan x = 0$. Hence or otherwise integrate using Monge's method

(10 marks)

QUESTION FOUR (20 MARKS)

a) Verify that each of the following equations has the indicated solutions

i. $u_{xx} - 3u_{xy} + 2u_{yy} = 0; u = f(x+y) + g(2x+y)$ (4 marks)

ii. $u_t = \alpha u_{xx}; u = e^{-\alpha t}\sin x$ (4 marks)

b) Obtain the integral of $q^2 r - 2pqs + p^2 t = 0$ in the form $y + xf(z) = F(z)$

(12 marks)

QUESTION FIVE (20 MARKS)

a) Solve the following equations

i. $u_{xy} = u_x$ (4 marks)

ii. $u_{xx} = 2xy; u(0,y) = y^2, u_x(0,y) = y$ (6 marks)

b) Find the D'Alembert's solution of the homogenous one-dimension wave equation $u_{xx} = \frac{1}{c^2}u_{tt}$ given that $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$ (10 marks)