



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATIONS FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCES)

SAC 302: FINANCIAL MATHEMATICS I

DATE: 26/7/2023

TIME:2:00-4:00 P.M

INSTRUCTIONS TO CANDIDATES.

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) An economist's model of interest rates indicates that the n -year spot rate of interest is $0.1(1 + e^{-0.1n})^{-1}$. According to this model, what is the price of a 10-year zero-coupon bond redeemable at par? (2 marks)
- b) A ten-year zero-coupon bond is issued on 1 February 2001 at a price of £79%. On 1 February 2003 an investor entered into forward contract to buy £1,000 nominal of the bond in 5 years' time. The price of the bond was £83% on 1 February 2003 and £92% on 1 February 2008. Calculate the profit or loss made by the investor on 1 February 2008 if the risk-free force of interest was 3% pa. (3 marks)
- c) A loan of nominal amount £100,000 is to be issued bearing coupons payable quarterly in arrears at a rate of 5% per annum. Capital is to be redeemed at 103% on a single coupon date between 15 and 20 years after the date of issue, inclusive. The date of redemption is at the option of the borrower. An investor who is liable to income tax at 20% and capital gains tax of 25% wishes to purchase the entire loan at the date of issue. Calculate the price which the investor should pay to ensure a net effective yield of at least 4% per annum (3 marks)

- d) Consider a fixed interest security that pays coupons of 10% at the end of each year and is redeemable at par at the end of the third year. Calculate (using an effective interest rate of 8% pa) the:
- i. volatility of the cash flow (2 marks)
 - ii. discounted mean term of the cash flows (2 marks)
 - iii. convexity of the cash flows. (2 marks)
- e) An insurance company has a continuous payment stream of liabilities to meet over the coming 20 years. The payment stream will be at a rate of £10 million per annum throughout the period. Calculate the duration of the continuous payment stream at a rate of interest of 4% per annum effective. (4 marks)

QUESTION TWO (20 MARKS)

- a) A 9-month forward contract is issued on 1 April 2004 on a stock with a price of £7 per share. Dividends of 50p per share are expected after 2 and 8 months.
- i. Assuming a risk-free effective rate of interest of 6% per annum and no arbitrage, calculate the forward price. (3 marks)
 - ii. Calculate the value of the forward contract on 1 May 2004 if the new price of the stock is £7.20 per share. (3 marks)
- b) An investor purchases a bond, redeemable at par, which pays half-yearly coupons at a rate of 8% per annum. There are 8 days until the next coupon payment and the bond is ex-dividend. The bond has 7 years to maturity after the next coupon payment. Calculate the purchase price to provide a yield to maturity of 6% per annum effective. (3 marks)
- c) The 1, 2, and 3-year spot rates are 3.5%, 4% and 3.7% respectively. The 2-year forward rate from time 3 is 5% and the 1-year forward rate from time 4 is 4.9%. Calculate the:
- i. 3-year forward rate from time 1 (2 marks)
 - ii. present value at time 0 of payments of £100 at times 3, 4 and 5 (2 marks)

QUESTION THREE (20 MARKS)

- a) A fixed interest stock is redeemable at 106% in 15 years' time and pays interest at 9% pa payable half-yearly in arrears.
- i) Calculate the price an investor should pay to obtain a gross redemption yield of 9% pa. (4 marks)
Instead of purchasing the stock, the investor decides to agree a forward contract to buy the security in six years' time, immediately after the coupon payment then due.
 - ii) Calculate the forward price based on a risk-free rate of return of 6% pa effective and no arbitrage. The current price of the stock is that calculated in part (i). (4 marks)
Three years later, the price of the security is such that the gross redemption yield is still 9%. Calculate the,
 - iii) value of the forward contract if the risk-free yield has not changed. (8 marks)
 - iv) yield obtained if the investor sold the forward contract after three years (4 marks)

QUESTION FOUR (20 MARKS)

- a) An insurance company has liabilities of £10 million due in 10 years' time and £20 million due in 15 years' time, and assets consisting of two zero-coupon bonds, one paying £7.404 million in 2 years' time and the other paying £31.834 million in 25 years' time. The current interest rate is 7% per annum effective.
- i. Show that Redington's first two conditions for immunization against small changes in the rate of interest are satisfied. (7 marks)
 - ii. Determine the profit or loss, expressed as a present value, that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective. (4 marks)
 - iii. Explain how you might have anticipated, before making the calculation in (ii), whether the result would be a profit or loss. (3 marks)
- b) A stock with a term of $9\frac{1}{2}$ years has a coupon of 5% pa payable half-yearly in arrears and is redeemable at 105%. An investor who is not subject to tax purchases the stock at 85 per 100 nominal. Calculate the yield obtained by the investor. (6 marks)

QUESTION FIVE (20 MARKS)

A stochastic interest rate model assumes that the annual growth factors for each future year are independently distributed lognormal random variables with parameters $\mu = 0.05$ and $\sigma^2 = 0.01$

- i. Calculate the mean and standard deviation of the annual rate of return. (6 marks)
- ii. Calculate the median, upper and lower quartiles of the distribution of the annual rates of return. (6 marks)
- iii. A single payment of £10,000 must be made at the end of 5 years. Calculate the probability that a single initial investment of £7,250 will be sufficient to meet the liability for this payment (4 marks)
- iv. State, with reasons, whether the probability calculated in (c) would be greater or smaller if the interest rate model assumed an unknown interest rate that was constant in all future years (with the same basic distribution), rather than assuming that the rates for each year are independent. [3 marks] You are given that the mode of a lognormal distribution is at $x = e^{u-\sigma^2}$ (4 marks)