MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 306: LIFE ASSURANCE

DATE:25/7/2023   **TIME:2:00-4:00 P.M**

INSTRUCTIONS TO CANDIDATES

> **Answer Question One and Any Other Two Questions**

QUESTION ONE (COMPULSORY) (30 MARKS)

a)   A special whole life assurance policy is issued on a life aged 50 exact.
Under this policy the sum assured, payable at the end of the year of death, is 1 unit for the first 10 years decreasing to 0.75 units thereafter

         i)   Calculate the expected present value of the benefit.   (3 marks)

         ii)   Determine the variance of the present value of the benefit.   (3 marks)

               Basis:
               Mortality AM92 Ultimate
               Interest 4% per annum

b)   An assurance policy provides a benefit of 1 payable immediately on the death of the last survivor of a male life aged 55 exact and a female life aged 50 exact.

      Determine:

         i)   the expected present value of this policy   (3 marks)

         ii)   the variance of the present value of this policy.   (3 marks)

               Basis:

               Force of mortality Male life – a constant force of 0.03   Female life – a constant force of 0.02 Force of interest 4%

c)   A company is about to establish a pension scheme that will provide retirement benefits to its members of n/80ths of final pensionable salary at age 65 or on earlier ill health,

where n is the total number of years of service to retirement. Age retirement in normal health follows the principles in the Pension Scheme Table for age retirement functions in the Formulae and Tables for Actuarial Examinations. Final pensionable salary is the average salary in the three years before retirement.

An employee joins the scheme aged 45 exact and is granted exactly 15 years of past service. The employee's salary in the year before joining was 35,000. Calculate the present value of benefits for this member (including future service. (6 marks

d) Explain three principles of insurance (6 marks)

e) Under a 20-year policy issued by a life insurance company, the benefit payable on death, at the end of the year of death, is a return of premiums paid without interest.

A premium of 631 is payable annually in advance, throughout the term of the policy. The following information is available for a policy in force at the start of the 19th year:

Reserves at the start of the year, 18V : 17,095

Reserves at the end of the year per survivor, 19V : 18,510

Probability of death during the year: 0.015

Rate of interest earned: 4.5% per annum

Determine the profit which is expected to emerge at the end of the 19th year for each policy in force at the start of that year. Ignore expenses and all decrements other than death. (6 marks)

QUESTION TWO (20 MARKS)

A life insurance company issues a 3-year guaranteed bonus endowment assurance policy to a life aged 62 exact with a basic sum assured of 75,000. The basic sum assured, together with any attaching bonuses, is payable at the end of the year of death or maturity if earlier.

Level premiums are payable annually in advance throughout the term of the policy or until earlier death.

Simple annual bonuses are added at the beginning of each policy year (i.e. the death benefit does include any bonus relating to the policy year of death).

The company uses the following basis for carrying out profit tests of this policy:

Mortality   AM92 Ultimate
Withdrawals   Ignore
Interest earned   5% per annum on cash flows
Expenses   Initial 15% of the first premium
                 Renewal 5% of subsequent premiums
Bonuses Simple bonus of 4% of basic sum assured per annum
The company holds net premium reserves for the policy using the following basis:

Mortality   AM92 Ultimate
Interest   4% per annum

a)   Calculate the net premium reserve for the policy at policy duration $t = 1$ and
      $t = 2$ years immediately before the premium then due.   (10 marks)

b)   Determine the annual premium required for the policy to achieve an internal
      rate of return of 6% per annum to the company.   (10 marks)

QUESTIONS THREE (20 MARKS)

a)   Describe, in the context of underwriting for life insurance, the following selection
      processes:
      i)   adverse selection   (3 marks)
      ii)   spurious selection   (3 marks)
      iii)   Describe an example of each selection process in part (i).   (2 marks)

b)   A life insurance policy for a male life aged 55 exact provides the following benefits:

      50,000 payable immediately on his death, if this occurs before the age of 65 exact.

      On survival to age 65 exact, a refund of 25% of total premiums paid without interest.

      On death of the male at any time, a pension of 5,000 per annum is payable monthly in
      advance to his widow (who is 5 years younger than him) for the remainder of her life,
      should she survive him. (This benefit is available throughout the lifetime of the male.)

      The policy is funded by premiums payable annually in advance for five years, or until
      the death of the male life, if earlier.

      Basis:

      Male mortality PMA92C20 Female mortality PFA92C20 Rate of interest 4 % per
      annum   Expenses Nil

      Calculate, showing all your workings, the premium for this policy.   (12 marks)

QUESTION FOUR (20 MARKS)

a) A 25-year "double" endowment assurance policy is issued to a group of lives aged 40 exact. Each policy provides a sum assured of 25,000 payable at the end of the year of death or 50,000 payable if the life survives until the maturity date.

Premiums are payable annually in advance throughout the term of the policy or until earlier death.

The following information has been provided:
Number of deaths during the 17th policy year: 24 Number of policies in force at the end of the 17th policy year: 5,350

    i) Calculate, showing all your workings, the profit or loss for the group arising from mortality in the 17th policy year. (10 marks)

    ii) Comment on your result. (2 marks)

Basis:

Mortality AM92 Select

Rate of interest   4% per annum

Expenses Ignore

b) A pension scheme provides a pension of 1/40 of final pensionable salary for each year of service, limited to a maximum of 2/3 of final pensionable salary, upon retirement at age 65. No other retirement age is allowed.

Final pensionable salary is defined as average annual salary over the 3 years immediately preceding retirement.

A member is now aged exactly 45 and has 16 years of past service. He earned £40,000 in the previous 12 months.

Calculate the expected present value now of this member's expected total pension on retirement.

    Basis:

        PEN Tables in the Formulae and Tables for Actuarial Examinations
        Interest 4% per annum (8 marks)

QUESTION FIVE (20 MARKS)

A life assurance company issues a policy which provides a three-year temporary annuity of £15,000 per annum payable annually in arrear to a male life aged 65 exact. The single premium payable at outset on the policy is £42,000.

The company uses the following basis to profit test the policy:

Mortality   PMA92C20

Interest earned on cash flow   and reserves 5% per annum

Initial commission   1% of the single premium

Initial expenses   £350

Renewal expenses   £55 per annuity payment which is assumed to increase by 3% per annum from inception of the policy

Risk discount rate 7% per annum

In addition, the company establishes reserves on the policy at the beginning and end of each policy year where:

$$tV = 15000 \times (3-t) \text{ for } t=1 \text{ and } 2$$

$$tV = 0 \text{ otherwise}$$

i)   Calculate the net present value of the expected profits on the policy: (13 marks)
   a)   allowing for reserves
   b)   ignoring reserves

ii)   Briefly comment on the reason for the difference in the two values calculated in part (i).   (3 marks)

iii)   Describe briefly how the net present value calculated in part (i)(a) and part (i)(b) would change if a risk discount rate of 4% per annum had been used   (instead of 7% per annum) and state the reasons for the difference.   (4 marks