MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 401: TOPOLOGY II

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer ALL the questions ONE and ANY TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

a) Define the following terms
   i) Finite intersection property
   ii) Sequentially compact
   iii) Compact spaces
   iv) Separated sets
   v) Locally compact sets (5 marks)

b) Prove that the open interval $A = (0, \frac{1}{2})$ on the real line $\mathbb{R}$ with the usual topology is not compact. (5 marks)

c) Consider the following topology on $X = \{a, b, c, d, e\}$ $\rho = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}, \{c\}\}$. Now $A = \{a, d, e\}$. Show that A is disconnected. (5 marks)

d) Prove that if X is sequentially compact then it is countably compact. (5 marks)

e) Prove that a set is disconnected if and only if it is not a union of two non-empty separated sets. (5 marks)

f) Prove that if X is compact then it is countably compact. (5 marks)

QUESTION TWO 20 MARKS

a) Prove that if A and B are disjoint compact subsets of Hausdorff spaces X. Then ∃ open set G and H such that $A \subset G$ and $B \subset H$ and $G \cap H = \emptyset$ (6 marks)

b) Let $\mathbb{Z}$ be the set of integers. Is it sequentially compact. (3 marks)

c) Let $G \cap H$ be a disconnection of A. show that $G \cap A$ and $H \cap A$ are separated. (5 marks)

d) Let $G \cup H$ be a disconnection of A and B be a connected subset of A. Show that $B \cap G = \emptyset$ or $B \cap H = \emptyset$. (6 marks)

QUESTION THREE 20 MARKS

a) Show that an infinite subset A of a discrete space X is not compact (7 marks)

b) Prove that a closed subset F of a compact set X is also compact. (7 marks)

c) Prove that a topological space X is compact if and only if $\{F_i\}$ of closed subsets of X satisfies the finite intersection property. (6 marks)

QUESTION FOUR 20 MARKS

a) Define hereditary as used in topological spaces (2 marks)

b) $T_{0\ and}\ T_1$ spaces are hereditary prove (6 marks)

c) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (6 marks)

d) Prove that if $X, \rho)$is a topological space which is a T_2. Then every convergent sequence of points of X has a unique limit. (6 marks)

QUESTION FIVE 20 MARKS

a) Prove that every compact subset of a Housdorff space is closed. (5 marks)

b) Show that if A and B are non-empty separated sets. Then, $A \cup B$ is disconnected (5 marks)

c) Show that every metric space is hausdorff space (5 marks)

d) Prove that if F is closed subset of a compact space X, then F is also compact. (5 marks)