MACHAKOS UNIVERSITY

University Examinations 2022/2023

SCHOOL OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERING
FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGRE OF BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

EMM 403: SOLID AND STRUCTURAL MECHANICS 111

DATE: **TIME: 2 HOURS**

INSTRUCTIONS

This paper contains FIVE questions

Question ONE is **compulsory** and carries 30 Marks.

Questions TWO – FIVE carries 20 Marks each. **Answer any Two.**

QUESTION ONE (COMPULSORY) (30 MARKS)

a) i) What is a thick-walled pressure vessel? (1 mark)

ii) A steel cylinder with internal diameter 160 mm and external diameter 320 mm. If it is subject to an internal pressure of 150 MPa. Determine, the maximum shearing stress in the cylinder. (3 marks)

b) Derive the Euler's equation that can be used to determine the crippling load of the column loaded as shown in Fig Q 1 b ii). (7 marks)

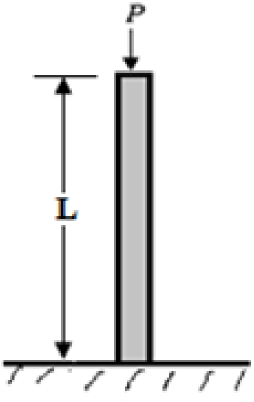

Fig. Q 1 b)

c) The cantilever beam AB supports a uniformly distributed load w, Fig. Q1 c). Determine the deflection and slope at A using Castigiliano's theorem. (6 marks)

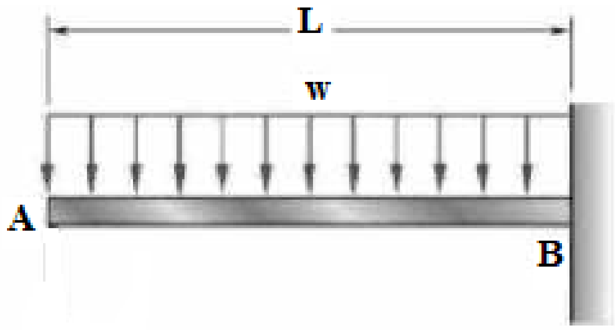

Fig. Q1 c)

d) i) Starting with the following equation;

$$\sigma_\theta - \frac{1}{t}\frac{d}{dr}(\sigma_r . t . r) - \rho\omega^2 r^2 = 0$$

show that for a constant strength disc of varying thickness, the thickness at any given radius is given by;

$$t_x = t_1 e^{-\frac{\rho\omega^2}{2\sigma_y}({r_x}^2 - {r_1}^2)}$$

Where;

t_1- thickness on the surface of the disc

r_1- radius of the disc

ρ - density of the disc

σ_y - yield stress

ω - angular velocity of the disc

r_x - radius where t_x is being determined (5 marks)

ii) A thin disc of mass 50 kg, radius 60 mm and thickness 25 mm rotates at a uniform angular speed of 70 rad/s. If the disc is fabricated from a material whose Poisson's ratio is 0.25, determine the radial and circumferential stresses developed at mid radius of the disc when in rotation. (3 marks)

e) A simply supported rectangular plate has sides $a \; x \; b$ and thickness t. The plate carries a uniformly distributed load P per unit area over the entire surface. Determine an expression for the maximum bending stress in the plate. (5 marks)

QUESTION TWO (20 MARKS)

a) Fig Q2. a) Shows a thick-walled cylinder with internal and external diameters a and b respectively carrying a fluid under internal pressure p_i and subjected to atmospheric pressure p_o.

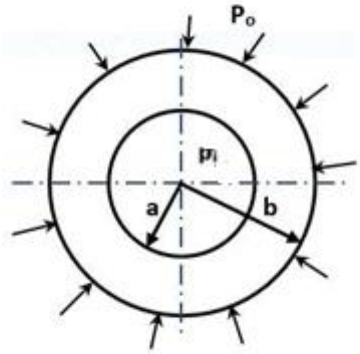

Fig Q2. a)

Derive the Lames equation that can be used to determine the tangential and radial stresses for a thick-walled pressure vessel. (12 marks)

b) A pressure vessel of 0.3 m internal radius, 0.4 m external radius and 0.9 m long, with closed ends is to be subjected to a hydraulic test of 20 N/mm^2. Calculate the change of internal and external diameters. E = 200 GPa and ν = 0.3 (8 marks)

QUESTION THREE (20 MARKS)

a) Fig. Q3 a) shows an initially curved pin-ended column of length L. If the column is further bent by a force P, show that maximum bending moment in the beam will be given by;

$M = PY\left[\frac{P_{cr}}{P_{cr}-P}\right]$ (12 marks)

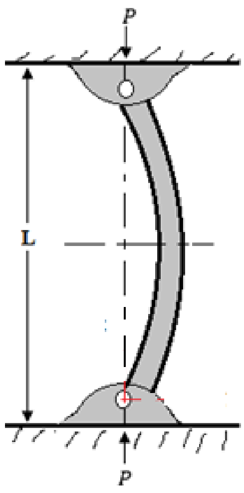

Fig. Q3 a)

Assume that the initial curvature is harmonic; $y_o = Y \sin\frac{\pi x}{L}$

b) An aluminium column is fixed at one end at the bottom and free at the top. Determine the minimum value of P if a factor of safety of 2 is desired. (Take L = 5 m, E = 70 GPa, $\sigma_y = 200\ \mathrm{MPa}, A = 7.5 * 10^{-3}\ \mathrm{m}^2, I_x = 6.13 * 10^{-5}\ \mathrm{m}^4, I_y = 2.32 * 10^{-5}\ \mathrm{m}^4$) (8 marks)

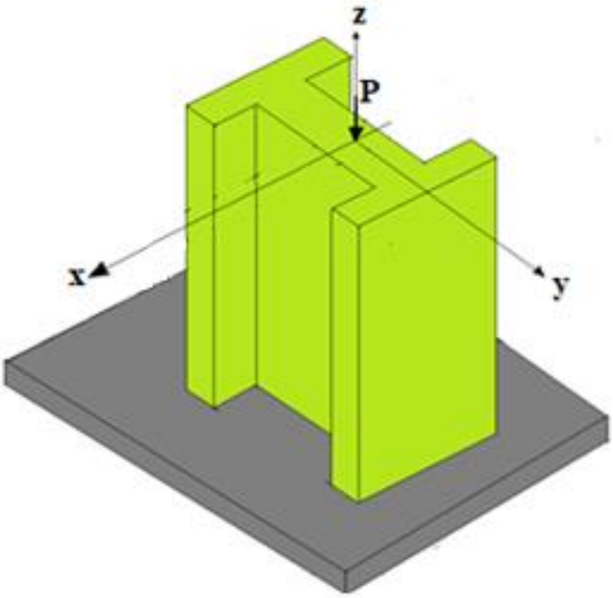

Fig. Q3 b)

QUESTION FOUR (20 MARKS)

Fig. Q 4) shows a circular plate with simply supported (edges) and radius ***a*** subjected to a uniform pressure ***P*** or load ***P*** per unit area.

a) Starting from $r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{1}{D}\int_0^r Prdr$, derive an equation for deflection (***w***) of the plate and hence determine the maximum deflection in the plate. (12 marks)

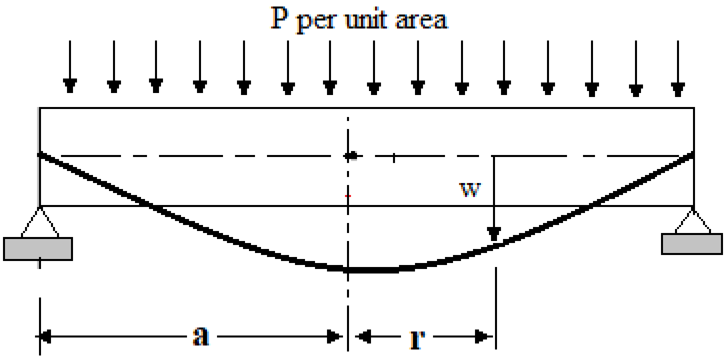

Fig. Q 4)

b) Determine the maximum bending stress in the plate in terms of ***P***, thickness ***t*** and plate radius ***a***. (8 marks)

QUESTION FIVE (20 MARKS)

a) The general equations for determination of circumferential and radial stresses of a thin disc rotating at an angular speed of ω are;

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$$

$$\sigma_r = A - \frac{B}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

where A and B are constants to be determined from boundary conditions for the particular problem.

Consider a circular disc of uniform thickness, internal radius R_i and outer raius R_o, rotating at a uniform angular speed ω, determine the expressions for maximum and mimium circumferential and radius stresses and hence sketch the stresses variations in the disc.

(12 marks)

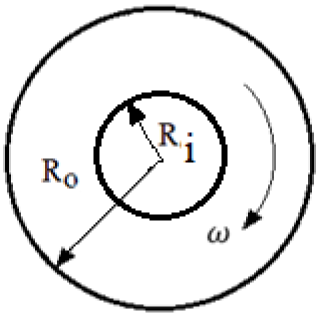

Fig. Q 5a)

b) A constant strength disc of diameter 950 mm and peripheral thickness 30 mm rotates at a constant speed of 300π rad/s. The maximum stress, σ_{max}, at any point of the disc is not to exceed 900 MN/m^2. Starting from the equilibrium equation;

$$\sigma_\theta - \frac{1}{t}\frac{d}{dr}(r.t.\sigma_r) - \rho\omega^2 r^2 = 0$$

determine the minimum thickness of the disc at the axis of rotation.

Assume density of the disc material is $\rho = 7.83 \text{ x } 10^3$ kg/m^3 (8 marks)