



# MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF BUSINESS, ECONOMICS AND HOSPITALITY AND TOURISM

MANAGEMENT

DEPARTMENT OF ECONOMICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF ECONOMICS

EES 800: ADVANCED MICROECONOMICS I

DATE:

TIME:

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**INSTRUCTIONS: Answer Question ONE and any other THREE questions**

## QUESTION ONE (COMPULSORY 30 MARKS)

- a) Clearly distinguish between the following pairs of concepts.
- (i) Hotelling's Lemma and Shepherds Lemma (2 marks)
  - (ii) Compensated and Uncompensated Demand functions (2 marks)
  - (iii) Price Leadership and Quantity Leadership (2 marks)

- b) Given the following Cobb-Douglas production of a firm

$Y = AL^\alpha K^\beta$  where Y is the output, L and K are units of labour and capital

- (i) Explain the economic meaning of the parameters A,  $\alpha$  and  $\beta$  (3 marks)
- (ii) Show the conditions under which the function will exhibit each of the three forms of global returns to scale (3 marks)
- (iii) Compute the elasticity of scale (overall elasticity of output) for the function (2 marks)

- c) In a duopoly market, the inverse market demand and cost functions of the firms are expressed as follows.

$$P(q) = 500 - q \text{ where } q = q_1 + q_2$$

$$C(q_1) = 2q_1^2$$

$$C(q_2) = 50q_2$$

- (i) Determine the Stackelberg equilibrium given that firm 1 is the quantity leader (4 marks)
- (ii) Determine the Cournot equilibrium (4 marks)
- (iii) Comment on how the Stackelberg equilibrium compare with the Cournot equilibrium above (2 marks)
- d) The indirect function of a certain household is expressed as follows.

$$V(p, m) = \frac{m^2}{9p_1p_2}$$

- (i) Derive the corresponding expenditure function and test its properties (3 marks)
- (ii) Derive the corresponding Hicksian demand functions. (2 marks)

### QUESTION TWO (12 MARKS)

- a) The production function of the company is defined as follows

$$Y = x_1^{0.3} x_2^{0.6}$$

where Y is the output and  $x_1$  and  $x_2$  are units of inputs used for production. The prices per unit of inputs  $x_1$  and  $x_2$  are  $w_1$  and  $w_2$  respectively

- i) Derive the corresponding profit function for the firm (3 marks)
- ii) Confirm the legitimacy of the profit function derived in (i) above (3 marks)
- b) The utility function of a certain household is expressed as follows:

$$U = 2q_1^2 q_2^2$$

where  $q_1$  and  $q_2$  are quantities of goods 1 and 2 consumed.

Determine the following

- (i) the corresponding expenditure function and test its properties (3 marks)
- (ii) Determine the Hicksian compensated demand functions for both goods (3 marks)

### QUESTION THREE (12 MARKS)

- a) A certain firm seeks to minimize its cost of production subject to producing a given level of output. The production function for the firm is expressed as follows:

$$Q = L^{\frac{1}{4}} K^{\frac{3}{4}}$$

where  $L$  and  $K$  are units of labour and capital used in production.

- i) Determine the conditional factor demand functions and the associated cost function of the firm (5 marks)
  - ii) Illustrate the properties of a legitimate cost function using the cost function obtained above (4 marks)
- b) Find the elasticity of substitution for the following general CES technology

$$f(x_1, x_2) = A(\alpha x_1^{-m} + \beta x_2^{-m})^{-\frac{1}{m}}$$

(3 marks)

### QUESTION FOUR (12 MARKS)

- a) A steel manufacturing company uses two inputs  $X_1$  and  $X_2$  for production of its output. The prices per unit of input is  $W_1$  and  $W_2$  for  $X_1$  and  $X_2$  respectively. The cost function of the company is defined as follows.

$$C(W_1, W_2, Q) = 18W_1^{1/3} W_2^{2/3} Q$$

where  $Q$  is the output,  $W_1$  and  $W_2$  are the prices of two inputs,  $X_1$  and  $X_2$  respectively

Determine the production function of the company. (6 marks)

- b) Using the function derived above explain the characteristics of a legitimate production function (6 marks)

**QUESTION FIVE (12 MARKS)**

The utility function of a certain household is expressed as follows:

$$U = 4X_1^2 X_2^2$$

- a) Derive Marshallian demand functions for goods 1 and 2 and tests its properties (6 marks)
- b) The prices per unit for the two composite commodities  $X_1$  and  $X_2$  are \$ 3 and \$ 4 respectively. The household has a monthly budgetary allocation of \$ 1200 for the two commodities. Determine the following
- i) Quantities of each good that the household should consume to maximize its utility (2 marks)
  - ii) The second order condition and interpret it (2 marks)
  - iii) The maximum utility attainable (2 marks)